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Approaches for Modeling of Shopping Mall Evacuation

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***Abstract*— This paper discusses mathematical and simulation models that could be applied to shopping mall evacuation problems. The so-called macroscopic approaches are based on dynamic network flow models. The maximum dynamic flow problem and the quickest flow problem are formulated. The so-called microscopic approaches are based on multi-agent systems. The possibilities for realistic reproduction of the processes are considered. On this basis, simple building layout is analyzed.**

Keywords—quickest flow problem, maximum dynamic flow problem, multi-agent systems, shopping mall evacuation.

Introduction

Building evacuation is the process of leading people out of a building to a safe place in the shortest possible time and the safest possible manner due to the occurrence of a hazardous event. Certain crisis situations allow for a priori estimation of evacuation time, comparing it with the threat propagation time and conducting a risk analysis. In such cases, time and potential risk are key indicators.

The evacuation time consists of three components:

- Perception and awareness time: The time required for people to recognize a dangerous situation. This time depends on the reliability of the alarm system and familiarity of evacuees with emergency signals.
- Decision-making time: The time to decide on an action. It depends on the level of evacuees training.
- Movement time to the safe zone (egress time): Depends on planned evacuation procedures, architecture, physical characteristics, and human behavior under stress.

The first two components primarily depend on organizational measures and behavioral patterns, making them difficult to reproduce analytically. Therefore, most evacuation models aim to determine the movement time to the safe zone. Factors such as the number and distribution of visitors and the throughput capacity of various sections during the process are also investigated.

Mathematical or simulation models are used for evacuation modeling; the former are typically called macromodels, while the latter are micromodels [1]. Macromodels are designed to estimate the lower bound of evacuation time. They do not account for individual differences in behavior, treating people as a homogeneous group with averaged characteristics. Modeling is reduced to formulating and solving network problems with dynamic flows. Micromodels, on the other hand, account for individual characteristics and the impact of interactions between evacuees on their movement.

The mathematical models were applied to a simplified layout of a shopping mall. The simulation model was developed within the NetLogo multi-agent systems environment. The model's capability to investigate the evacuation processes and the performance of its individual components has been verified.

Discrete Mathematical Models of Evacuation

To formulate optimization problems related to shopping mall evacuation, the building is represented as a directed static network. Evacuation over time is modeled using a dynamic network derived from this static one. Areas within the building with a significant occupancy are treated as source nodes in the static network. The safe zone outside the building is represented as a single sink node connected to the exits. In this sense, evacuation problems are modeled as network flow problems with multiple

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sources and a single sink. Each node has a capacity, which is the upper bound of people it can accommodate simultaneously. Each arc is characterized by two parameters: travel time and flow capacity. The former indicates the transition time between nodes, while the latter represents the maximum number of people per unit of time through a cross-section of the arc. These two parameters can be assumed constant or treated as time-variant throughout the evacuation process. If, for instance, the passage between two zones is obstructed at a certain moment, the capacity of the corresponding arc in the dynamic network should be set to zero.

Let $G = (N, A)$ be a static network with N the set of nodes and A the set of arcs. The travel times along an arc $(i, j) \in A$ are non-negative integers denoted by τ_{ij}. The dynamic network $G_T = (N_T, A_T)$ is a time-expanded version of the static network for T periods (steps), where $N_T := i(t) | i \in N; t = 0,1, \dots, T$. That is, each node in N corresponds to $T + 1$ nodes $i(t)$ in G_T, and the set A_T consists of movement arcs
$A_M := \{(i(t), j(t + \tau_{ij}) \mid (i, j) \in A; t + \tau_{ij} \leq T; t = 0,1, \dots, T\}$ and holdover arcs
$A_S := \{(i(t), i(t + 1) \mid i \in N; t = 0,1, \dots, T - 1\}$.

The capacity of the arcs A_M at time t is $c_{ij}(t)$, while the capacity of the arcs A_S is equal to the node capacity $c_i(t)$. The number of people in node i at $t = 0$ is q_i. Capacities are defined as non-negative integers.

Let $x_{ij}(t)$ denote the flow that departs from node i at time t and reaches node j at time $t + \tau_{ij}$. The flow present in node i at time t (remaining units in that node at time $t - 1$) is denoted by $y_i(t)$.

The arc travel times and the number of time periods (steps) T are determined by the selected basic unit θ (the duration of a single step). The total number of periods T is obtained by dividing the evacuation time by θ. The choice of the basic unit θ is a compromise between the accuracy of the results and the computational complexity.

For illustration, a simplified layout of a shopping mall, as shown in Fig. 1, is used. It features four shops, each with a single door, a corridor, and two exits.

The corresponding static network (Fig. 2) has been constructed, with the corridor divided into three zones.

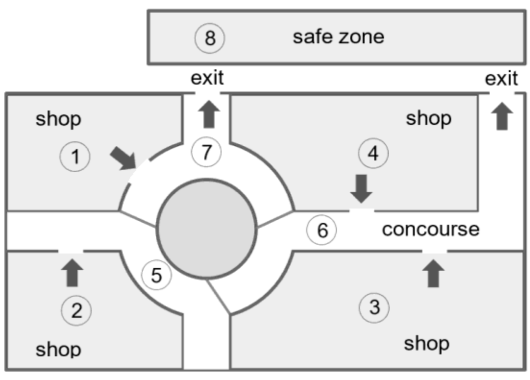

Fig. 1. Simplified building layout.

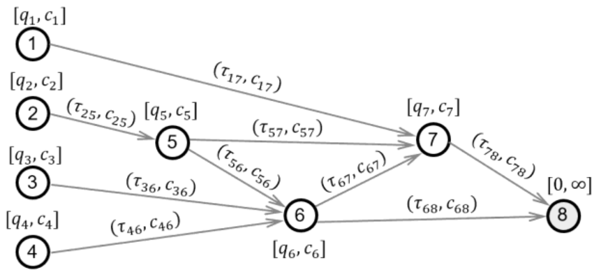

Fig. 2. Static network.

The network consists of eight nodes and nine arcs, where node 8 is the sink node, and the others are source nodes.

The dynamic network contains $T + 1$ copies of each source node and each sink node from the static network. A single super source and a single super sink node are added to it. The way the super source is connected to the actual source nodes depends on the specific problem being addressed. Dynamic flow problems can be treated as static flow problems within the dynamic network. In this framework, it is not necessary to assume that the input data remain constant over time. However, if this assumption holds, efficient solution algorithms can be utilized.

A. Average Evacuation Time Minimization Problem

Fig. 3 shows part of the dynamic network for this problem, based on the following input data: $\theta = 5$, $T = 5$;

- nodes

i	1	2	3	4	5	6	7	8
c_i	80	60	100	70	40	60	50	∞
q_i	50	20	60	10	20	30	25	0

- arcs

(i,j)	(1,7)	(2,5)	(3,6)	(4,6)	(5,6)	(5,7)	(6,7)	(6,8)	(7,8)
τ_{ij}	2	2	3	2	4	3	3	4	1
c_{ij}	15	15	15	15	15	15	15	15	15

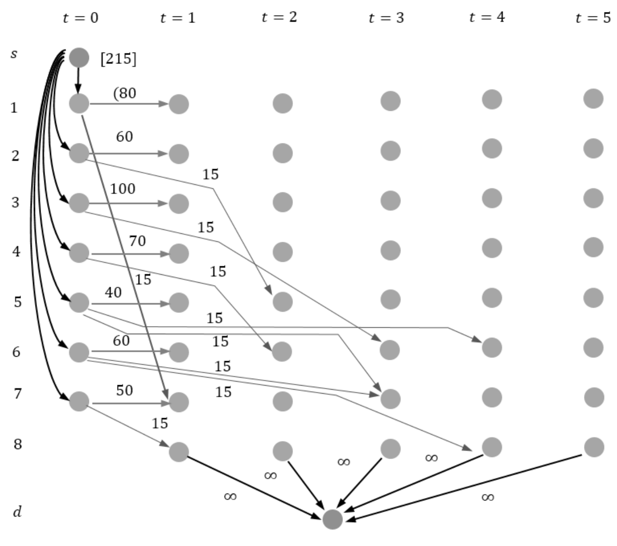

Fig. 3. Dynamic network.

The super source node s contains all the people in the building. Node s is connected only to the copies of the source nodes at $t = 0$. Arcs capacity is equal to the initial number of people in the respective nodes, i.e., $c_{s1} = 50, c_{s2} = 20$, and so on. All copies of the sink node 8 are connected to the super sink node d via arcs with infinite capacity. To avoid overcomplicating the figure, only the transition arcs $\left(i(0), j(\tau_{ij})\right)$, originating from nodes at $t = 0$, are shown. These arcs are repeated for $t = 1, 2, \dots, T - \tau_{ij}$, with capacities assigned according to the input data. Regarding holdover arcs, only those between the node copies at $t = 0$ and $t = 1$ are displayed; their capacity is equal to the respective node capacities. It should be noted that to solve the problem, the number of steps T must be significantly greater than 5. Furthermore, all node and arc copies that do not belong to any path between s and d are removed from the dynamic network.

Let $S \subset N$ be the set of source nodes in G, and $D \subset N$ be the set of sink nodes. The evacuation time for a given individual is equal to the time t at which they reach a node in D. This can be treated as the (turnstile) cost of the arcs $(i(t), d)$, $i \in D$; $t = 1,2, \dots, T$. Consequently, the average evacuation time is defined as:

$$\frac{\sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t)}{\sum_{i \in S} q_i}$$

Since the denominator is constant, the formulation of the optimization problem, assuming constant input data (c_i, c_{ij}, τ_{ij}), is as follows.

Find the minimum value of

$$Z = \sum_{t=0}^{T} \sum_{i \in D} t x_{id}(t), \tag{1}$$

subject to the following constraints:

$$x_{si}(0) = q_i, \forall i \in S \tag{2}$$

$$\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t) = \sum_{j \in S} q_i \tag{3}$$

$$y_i(t+1) = y_i(t) + \sum_{k \in p(i)} x_{ki}(t - \tau_{ki}) - \sum_{j \in s(i)} x_{ij}(t) \tag{4}$$

$$t = 0,1, \dots, T;\ \forall i \in N$$

$$y_i(0) = 0,\ \forall i \in N \tag{5}$$

$$y_i(t) = 0,\ \forall i \in D; t = 1, \dots, T$$

$$0 \le y_i(t) \le c_i,\ t = 1, \dots, T;\ i \in N - D; \tag{6}$$

$$0 \le x_{ij}(t) \le c_{ij}, t = 0, \dots, T - \tau_{ij}; \forall (i,j) \in A \tag{7}$$

where $p(i) \coloneqq \{j \mid (j,i) \in A\}$ and $s(i) \coloneqq \{j \mid (i,j) \in A\}$.

Constraints (4) define the net flow at the nodes of G_T, (5) impose restrictions on availability, while (6) and (7) represent the capacity constraints. To obtain the solution, any algorithm for the minimum cost flow problem is applied.

B. *Maximum Dynamic Flow (MDF) Problem*

This problem involves determining the maximum dynamic flow that reaches the sink node within a predefined number of steps (time horizon) T. In other words, it aims to determine the maximum number of people who can be evacuated from a given building within a specified time. The problem can be solved by applying any of the algorithms for finding the maximum static flow from node s to node d in the dynamic network G_T. When constructing this network, it should be noted that the super source s is connected to all copies of all source nodes in G via arcs with infinite capacity and zero travel time. Furthermore, holdover arcs can be omitted, as it has been proven that they are not utilized by the flow [1]. The mathematical formulation for this setup is as follows.

Find the maximum value of

$$v(T) \tag{8}$$

subject to the following constraints:

$$\sum_{t=0}^{T} \sum_{i \in S} x_{si}(t) = v(T) \tag{9}$$

$$\sum_{j \in s(i)} x_{ij}(t) - \sum_{k \in p(i)} x_{ki}(t - \tau_{ki}) = 0 \tag{10}$$

$$t = 0,1, \dots, T;\ i \ne s, d$$

$$\sum_{t=0}^{T} \sum_{i \in D} x_{id}(t) = v(T) \tag{11}$$

$$0 \le x_{ij}(t) = c_{ij}, t = 0, \dots, T - \tau_{ij}; \forall (i,j) \in A \tag{12}$$

Solving the MDF problem using the G_T network is justified only if the capacities and travel times vary over time. However, if these remain constant, an efficient method exists for generating the MDF from a static flow [2]. This approach utilizes a network G with a single source node s and a single sink node d. If necessary, the network model of the building is brought into this form by adding a super source. The algorithm consists of the following steps:

1) Solve the problem of finding a static flow f from the source to the sink node of G that maximizes the linear function

$$(T+1)v - \sum_{(i,j) \in A} \tau_{ij} x_{ij} \tag{13}$$

subject to the following conditions:

$$\sum_{i \in N} x_{si} = v$$

$$\sum_{j \in s(i)} x_{ij} - \sum_{k \in p(i)} x_{ki} = 0,\ i \ne s, d \tag{14}$$

$$- \sum_{i \in N} x_{id} = -v$$

$$0 \le x_{ij} = c_{ij} \tag{15}$$

This is essentially a minimum cost static flow problem. The maximum value of the function is equal to the magnitude of the MDF for T periods.

2) The MDF itself is found as follows:

- Decompose the static flow f with magnitude v into path flows between s and d. The following holds: $v = \sum_{i=1}^{l} v(P_i)$, where $v(P_i)$ is the magnitude of the flow along the i-th path.
- Each flow is repeated from $t = 0$ to $t = T + 1 - \tau(P_i)$, where $\tau(P_i)$ is the travel time along the i-th path (i.e., $v(P_i)$ units are sent at each step).

The dynamic flows obtained in this manner are called *temporally repeated flows* [1],[2].

Numerical analysis of the network from Fig. 2 shows that the magnitude of the MDF increases linearly for $T > 3$ by 30 units per step. This rate is determined by the total capacity of the arcs leading to node 8; thus, the throughput capacity of the building's exits is critical with respect to the maximum flow. It should be noted that this model assumes full utilization of each arc leading to the safe zone.

The MDF problem can be further refined to seek the maximum flow at each time step t. In this case, the objective function is redefined as $\sum_{t=1}^{T'} \sum_{i \in D} x_{id}(t), \forall T' = 1, \dots, T$.

C. Quickest Flow (QF) Problem

In this problem, the objective is to minimize the time $T(v)$ required for a given number of initial occupants v to exit the building, i.e., the time in which a dynamic flow of a specified magnitude v travels from the source node s to the sink node d of the network. Net flow and arc capacity constraints apply. In a sense, the QF problem is the inverse of the MDF problem. It has been proven that the function $v(T)$, which defines the change in MDF, is monotonically increasing. The maximum increment Δv is equal to the maximum flow in the static network. Thus, to find $T(v)$ for a given v, one must find the smallest integer T such that $v(T) \geq v$. This can be achieved, for example, by using binary search.

Polynomial-time solution algorithms for the problem were proposed in [3]. In [4], the QF problem is solved by means of an optimization problem with a fractional objective function defined over the static network.

AGENT-BASED SIMULATION MODEL OF EVACUATION

The workspace in NetLogo is referred to as 'world'. It is composed of 'patches'. For this model, the size of a single patch is defined as 1 sq. m. The simulation progresses in intervals called 'ticks', which are assumed to have a duration of 1 second. Individual patches can represent: a wall, an obstacle, an exit, a safe zone, empty space (white), etc. These elements are used to construct the map of the world. Fig. 4 shows the map of simplified shopping center at a certain stage of the evacuation process.

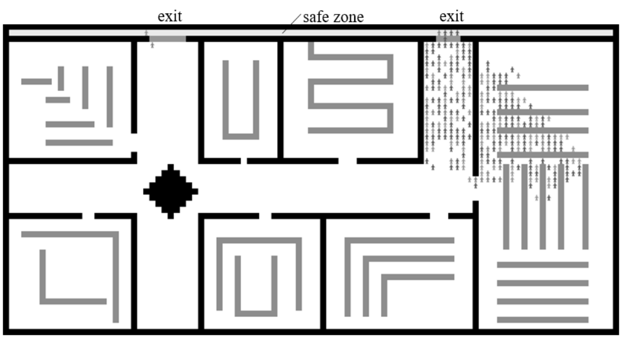

Fig. 4.

The model parameters are defined via a user interface and are categorized into two primary groups: initial state parameters and parameters influencing the evacuation process.

The initial state is determined by the number of occupants present in the shopping mall and their current positions within the facility at the onset of the evacuation. The occupancy count is treated as an input parameter, ranging from 40 to 3,800 individuals for this specific model. Their spatial distribution is generated randomly, following a uniform probability distribution.

Parameters affecting the evacuation process include individual movement speeds, the maximum occupancy density per "patch" (per sq. m), and the flow capacity of the exits. For this particular simulation experiment, the movement speed for all individuals within the shopping mall is fixed at 1 m/s.

The maximum number of people allowed in a single patch (per sq. m) is defined. This effectively sets the throughput capacity of the exits. This capacity is determined twice: first, by the number of patches when drawing the map, and second, by defining how many people can pass through one patch within 1 second.

In addition to input data, the interface dynamically displays real-time data regarding the evacuation process: the number of people remaining in the center at any given moment and the number of those who have reached the safe zone. The initial state of the population is generated first, and then the simulation begins.

The direction of movement for each individual is determined by a pre-generated distance map, which indicates the distance from every patch to the nearest exit. Fig. 5 shows the distance map of the modeled shopping center. Before taking each step, every person selects the adjacent patch with the smallest distance value to the exit. If multiple patches share the same minimum value, one is selected at random. A check is performed if the patch is 'free', meaning the occupancy limit has not been reached. If the patch is available, the person moves in that direction. However, if the patch is occupied, the other forward-facing patches (to the left and right of the chosen one) are checked; if any of them is free, the movement is redirected toward it. If all three patches are occupied, the individual waits for the current tick without moving.

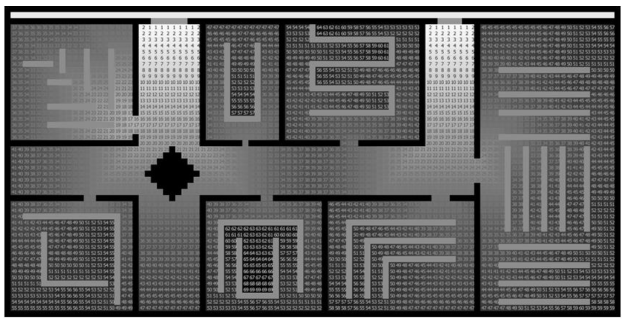

Fig. 5.

For this specific study, a 'world' with dimensions of 100 x 50 m was created. The layout of walls, obstacles, exits, and the safe zone is visible on the map on Fig. 4. A series of numerical experiments were conducted at two different exit throughput capacities: for the left exit- 6 persons per second and 2 persons per second; and for the right exit- 4 persons per second and 2 persons per second. For the first series of experiments, the total throughput capacity of both exits is 10 people per second, while for the second series, it is 4 people per second. Simulation ends when the last person leaves the building.

The performance measure is the evacuation completion time, i.e., the moment when the last visitor has left the shopping center. For each combination, ten experiments were conducted, the results were averaged and are shown in Fig. 6. The horizontal axis represents the average number of people at the initial moment of the simulation, while the vertical axis shows the seconds within which the evacuation process was completed.

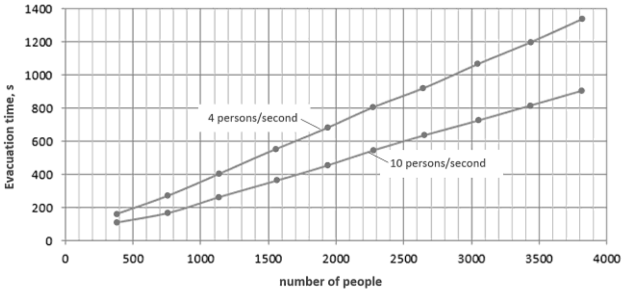

Fig. 6.

A small subset of the statistical data regarding the results obtained at an exit flow capacity of 10 people per second is presented in Table 1.

TABLE 1. STATISTICAL ANALYSIS OF EVACUATION TIME

Occupancy (Number of People)	Mean Value, s	Standard Deviation, s	Confidence Interval, s
750	175.4	6.52	± 3.79
1550	327.3	12.89	± 7.47
1950	448.2	10.52	± 6.1

The data in the table indicates that the dispersion around the calculated mean values is minimal, demonstrating that the obtained results are reliable for use as outcomes of the numerical experiment.

The first and most obvious conclusion is that the capacity of the exits significantly influences the evacuation time. The graph shows that the relationship between evacuation time and the number of people in the building is linear. Of particular interest is the zone between 400 and approximately 800 people at a throughput capacity of 10 people per second. It is evident that in this range, time increases considerably slower than it does for populations exceeding 800. This is due to the fact that when more than 800 people are in the shopping center, congestion begins to form at the exits (the right exit in Fig. 4). In other words, with fewer than 800 people in the building, the evacuation rate depends on the distances and the movement speed of the individuals, whereas at higher values, it is governed by the throughput capacity of the doors.

CONCLUSIONS AND FUTURE WORK

The process of building evacuation can be modeled through network problems, providing optimistic estimates of performance indicators. Given the structure of the static network in building evacuation, its maximum flow-and consequently, the increase in maximum dynamic flow per period-is determined by the capacity of the exits.

Average Evacuation Time Minimization Problem is the most suitable for developing evacuation plans and conducting sensitivity analyses of the solution relative to the input data. This model accounts for the initial number of people and node capacities, allowing for the use of time-varying travel times and capacities throughout the process. It has been proven that the flow which minimizes the average evacuation time also maximizes the number of people reaching the safe zone at each time step and minimizes the total evacuation time [1].

Discrete mathematical models can be further refined. The area subject to evacuation can be divided into priority zones based on the level of threat. Arc travel times can be treated as a function of time and flow density.

To account for the influence of behavioral and organizational factors on the evacuation process, simulation models should be utilized. Current trends in the field involve the use of hybrid models, which simulate both the behavior of evacuees and concurrent physical processes, such as the spread of fire and smoke.

The simulation model presented here is relatively simple, yet even in this form, it can approximate the evacuation process and outline directions for its improvement. There are numerous opportunities for further developing the model. For instance, a hazard zone could be added that expands over time, blocking paths to specific exits, causing injuries to individuals, and slowing their movement. The behavior of individual agents is also subject to further refinement.

Numerous software products have been developed for solving evacuation problems, based on either mathematical or simulation models [5],[6].

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REFERENCES

[1] H. W. Hamacher and S. A. Tjandra, "Mathematical modelling of evacuation problems: A state of the art," Fraunhofer ITWM, Kaiserslautern, Germany, Tech. Rep., 2001.

[2] L. R. Ford and D. R. Fulkerson, Flows in Networks. Princeton, NJ: Princeton University Press, 1962.

[3] R. E. Burkard, K. Dlaska, and B. Klinz, "The quickest flow problem," Methods and Models of Operations Research, vol. 37, pp. 31–58, 1993.

[4] P. Jaillet and L. Maokai, "On the quickest flow problem in dynamic networks: A parametric min-cost flow approach," Networks, vol. 65, no. 2, pp. 123–138, 2015, https://doi.org/10.1002/net.21574.

[5] E. D. Kuligowski and R. D. Peacock, A Review of Building Evacuation Models. Gaithersburg, MD: National Institute of Standards and Technology, 2005.

[6] S. Casadesús-Pursals, F. Garriga, and M. Pujol, “Building evacuation: Principles for the analysis of basic structures through dynamic flow networks,” Journal of Industrial Engineering and Management, vol. 6, no. 4, pp. 1081–1094, 2013, https://doi.org/10.3926/jiem.805.

[7] L. G. Chalmet, R. L. Francis, and P. B. Saunders, “Network models for building evacuation,” Management Science, vol. 28, no. 1, pp. 86–105, 1982, https://doi.org/10.1287/mnsc.28.1.86.

[8] S. A. Tjandra and H. W. Hamacher, “Dynamic network optimization with application to evacuation problems,” in Operations Research Proceedings, 2003, pp. 255–262.