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On the Vibrations of Multi-Cylinder Marine Diesel Generators

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***Abstract*— In modern ship power plants with electric propulsion motors, the main ship power system consists of powerful multi-cylinder diesel generators. The engine and the generator are mounted on a common frame, which is attached to the ship's hull through resilient (elastic) elements.**

Minimizing the vibrations induced in the ship's hull is achieved by refining the mechano-mathematical models. In the present work, this is performed using a mechano-mathematical model in which the diesel-generator-frame system is modeled as an elastic body with distributed parameters.

A model has been developed in which the ship's power plant undergoes vibrations in a single plane. The resulting system of differential equations describes the forced vibrations of the proposed dynamic model.

The natural frequencies of free vibrations have been obtained. Forced vibrations excited by the imbalance of the generator rotor are investigated.

Results from experimental studies of a nine-cylinder diesel generator are presented.

The presented theoretical and experimental results demonstrate the necessity of accounting for bending vibrations when investigating the overall vibrations of resiliently mounted multi-cylinder diesel generators.

Keywords*— *diesel generators; natural frequencies; oscillation.

I. Introduction

Marine diesel generators are mounted elastically to minimize the vibrations that excite the ship's hull. The vibration exciters are the unbalanced inertial forces of the diesel engine and the unbalance of the generator rotor. The vibrations are studied using mathematical models [1-3] with varying degrees of complexity. Typically, the object is modeled as a rigid body with one, two or six degrees of freedom. Attempts to minimize the weight of industrial units through various optimization procedures have led to the influence of the elasticity of the machine body on the vibration state [4-8]. The effect is most pronounced in powerful elastically mounted industrial fans. The frame on which the fan and engine are mounted performs bending vibrations [4, 6, 7, 8]. These vibrations can be of high intensity at resonance, despite the elastic mounting. The elasticity of the main ship engines in large ships [7] is a cause of excitation of vibrations because of the elastic anisotropy of the crankshaft. When the ship is loaded, the multi-cylinder engine is deformed because of the deformation of the ship hull. The misalignment of the bearings caused by the elastic deformation of the engine is the cause that leads to excitation of vibrations from the crankshaft. In modern ship power systems with rowing electric motors, the main ship power system consists of powerful multi-cylinder diesel generators. The engines have fully balanced inertial forces. The excitation of vibrations can be an imbalance of the generator rotor or inhomogeneity of the magnetic field. The engine and generator are mounted on a common frame, which is attached to the ship hull by elastic elements. Minimization of vibrations excited on the ship hull is achieved by improving the mechano-mathematical models. In the present work this is done through a mathematical model in which the system - diesel generator - frame is modeled as an elastic body (Fig. 1).

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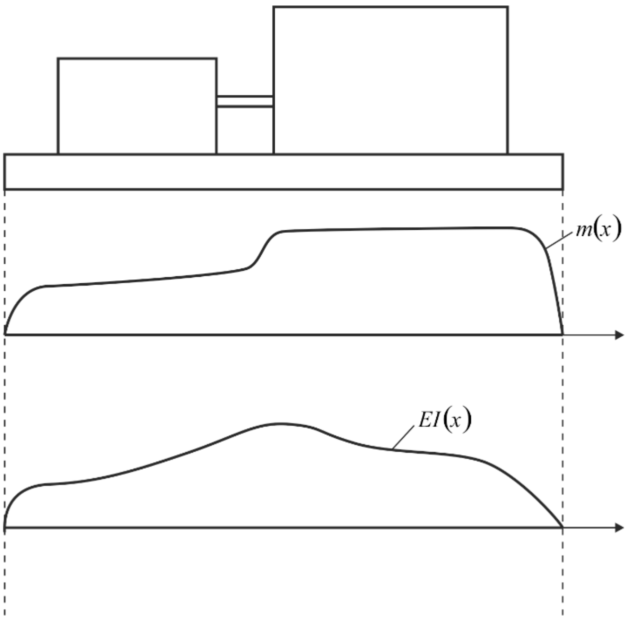

Fig.1. System frame.

The machine unit is a body with distributed parameters. The elastic properties are given by the function $EJ(x)$, and the mass ones through the function $m(x)$. C E is denoted by the modulus of elasticity, and by $J(x)$ the moment of inertia of the geometric sections.
In the model presented in Fig. 1, the elastic suspension is not represented. It consists of n - number of elastic elements (Fig. 2) with stiffness $C_i (i = 1, 2, \dots, n)$.
We will build a model in which the ship's power system performs vertical oscillations in one plane (Fig.2).

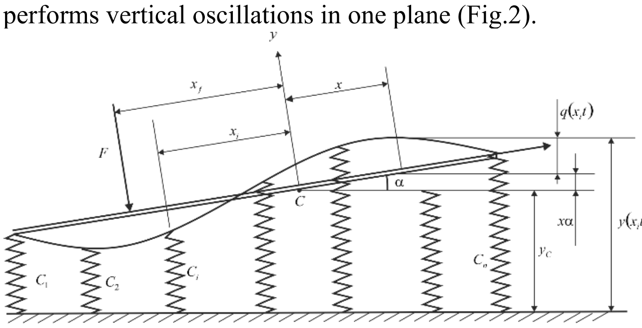

Fig.2. Vertical oscillations.

Oscillations of the center of mass C are marked with y_C, and the angular oscillations about an axis passing through the center of mass (orthogonal to the plane) are denoted by α. The elastic beam with distributed parameters $m(x)$ and $EJ(x)$ through which the diesel generator is modeled together with the frame performs oscillations, which are indicated by $q(x,t)$. With F is the force that excites oscillations as a result of unbalance of the rotor of the electric motor or inhomogeneity of the electromagnetic field. The force is applied in a cross section x_f.

The transverse displacement during vibrations of the beam with distributed parameters will be represented in the form
(1) $q(x,t) = \vartheta(x)\varphi(t)$
where $\vartheta(x)$ is the form of the free vibrations of the elastic beam, and $\varphi(t)$ is an unknown function of time. The displacement resulting from the oscillations of the cross-section with coordinate x we mean by $y(x,t)$ (Fig. 2). It is the result of the transverse vibrations of the beam as an elastic body.
$q(x,t)$ and the vibrations of the beam as a rigid body defined by $y_c(t)$ and $\alpha(t)$:
(2) $y(x,t) = q(x,t) + y_c(t) + x\alpha(t)$
Assuming generalized coordinates $\varphi(t), y_c(t), \alpha(t)$ we will compose the differential
equations describing oscillations, applying the Lagrange formalism
Kinetic energy has the form:
(3) $T = \frac{1}{2}\int_0^L \dot{y}^2(x,t)m(x)dx$
we substitute (2) into (3) and obtain
(4)
$$T = \frac{1}{2}M\dot{\varphi}^2 + \frac{1}{2}m\dot{y}_c^2 + \frac{1}{2}J_z\dot{\alpha}^2 + m_1\dot{\varphi}\dot{y}_c + m_2\dot{\varphi}\dot{\alpha} + m_3\dot{y}_c\dot{\alpha}$$
where:
(5)
(5.1) $M = \int_0^L \vartheta^2(x)m(x)dx$
(5.2) $J_z = \int_0^L x^2 m(x)dx$
(5.3) $m_1 = \int_0^L \vartheta(x)m(x)dx$
(5.4) $m_2 = \int_0^L \vartheta(x)xm(x)dx$
(5.5) $m_3 = \int_0^L xm(x)dx$
Potential energy has the form:
(6) $\Pi = \frac{1}{2}\int_0^L EJ(x)y''(x,t)dx + \frac{1}{2}\sum_{i=1}^n C_i y(x_i,t)^2$
here $y'' = \frac{d^2y}{dx^2}$
x_i - coordinate of i - elastic element of the suspension with stiffness C_i.
After substituting (2) into (6) we obtain
(7) $\Pi = \frac{1}{2}C\varphi^2 + \frac{1}{2}\sum_{i=1}^n C_i\left[\vartheta(x_i)\varphi + y_c + x\alpha\right]^2$
where
(8) $C = \int_0^L EJ(x)\vartheta''(x)^2 dx$

$$\vartheta'' = \frac{d^2\vartheta}{dx^2}$$

The possible work is determined by

$$\delta A = F\delta y = F\left[\vartheta(x_f)\delta\varphi + \delta y_c + x_f\delta\alpha\right]$$

from here we get the generalized forces

$$Q_\varphi = F(t)\vartheta(x_f), \quad Q_{y_c} = F(t), \quad Q_\alpha = F(t)x_f$$

Applying Lagrange's formalism, the equations are obtained:

(9)

$$\begin{array}{|l} M\ddot{\varphi} + m_1\ddot{y}_c + m_2\ddot{\alpha} + C\varphi + \sum C_i[\vartheta(x_i)\varphi + y_c + \\ +x_i\alpha]\vartheta(x_i) = F\vartheta(x_f) \\ m\ddot{y}_c + m_1\ddot{\varphi} + m_3\ddot{\alpha} + \sum C_i[\vartheta(x_i)\varphi + y_c + x_i\alpha] = F \\ J\ddot{\alpha} + m_2\ddot{\varphi} + m_3\ddot{y}_c + \sum C_i[\vartheta(x_i)\varphi + y_c + x_i\alpha]x_i = Fx_f \end{array}$$

The system of differential equations (9) describes the forced oscillations of the dynamic model in Fig. 2, through which the oscillations of the diesel generator are modeled. The function $\vartheta(x)$ defines the form of free oscillations of the elastic beam with distributed parameters. The principal vector of the vertical inertial forces must be equal to zero

(10) $\int_0^L m(x)\ddot{q}dx = 0$

The principal moment of inertia forces must also be equal to zero:

(11) $\int_0^L m(x)\ddot{q}xdx = 0$

After substituting (1) into (10) and (11) we obtain:

(12) $$\begin{aligned} &\int_0^L m(x)\vartheta(x)dx = 0 \\ &\int_0^L m(x)\vartheta(x)xdx = 0 \end{aligned}$$

It follows from this that for (5.3) and (5.4) we obtain

(13) $m_1 = 0, \quad m_2 = 0$

The coordinate system (XY) is with origin at the center of mass. It follows that for (5.5) is satisfied.

(14) $m_3 = \int_0^L xm(x)dx = X_c m = 0$

Based on (13) and (14), the system of differential equations (9) takes the form:

(15)

$$\begin{aligned} &M\ddot{\varphi} + C\varphi + \sum C_i[\vartheta(x_i)\varphi + y_c + x_i\alpha]\vartheta(x_i) = F\vartheta(x_f) \\ &m\ddot{y}_c + \sum C_i[\vartheta(x_i)\varphi + y_c + x_i\alpha] = F \\ &J\ddot{\alpha} + \sum C_i[\vartheta(x_i)\varphi + y_c + x_i\alpha]x_i = Fx_f \end{aligned}$$

The free (bending) vibrations of the elastic beam with distributed parameters are described by the equation

(16) $M\ddot{\varphi} + C\varphi = 0$

Free vibrations are of the form

(17) $\varphi = \Phi \sin p_c t$

Substituting (17) into (16) we obtain the frequency of free (bending) vibrations.

(18) $$p_c^2 = \frac{C}{M} = \frac{\int_0^L EJ(x)(\vartheta''(x))^2 dx}{\int_0^L m(x)\vartheta^2(x)dx}$$

This is the frequency of (bending) vibrations of the free (unsupported) beam.

The actual forms of the free vibrations are not known. For any given function $\vartheta(x)$ corresponds to a certain value of P_C. According to Rayleigh's theorem, the functional (18) is minimized for the true form of oscillation. The lowest (first) eigenfrequency corresponds to a two-node form of oscillation, the second to a three-node form (Fig. 3), and so on.

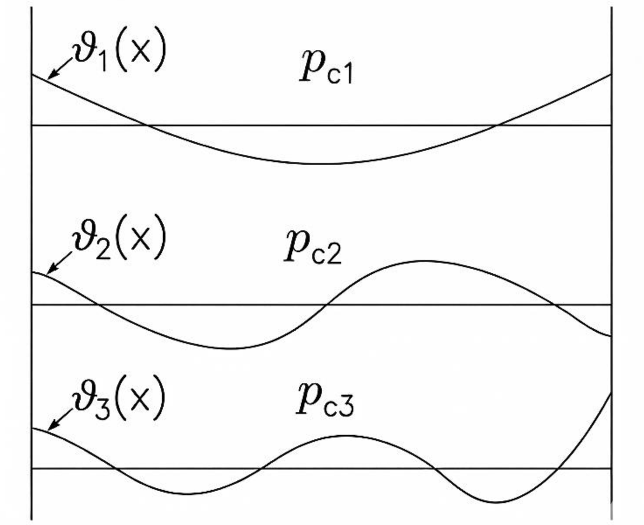

Fig.3. Node forms of oscillation.

The free (bending) vibrations of the elastically mounted beam are described by the equation (19)

$$M\ddot{\varphi} + \left[C + \sum C_i\vartheta^2(x_i)\right]\varphi = 0$$

From here we obtain the frequency of free (bending) vibrations of the elastically mounted beam

(20) $$p^2 = \frac{C + \sum C_i\vartheta^2(x_i)}{M} = p_c^2 + \frac{\sum C_i\vartheta^2(x_i)}{M}$$

The vibrations of the beam as a rigid body are described by the equations

(21) $$\begin{array}{|l} m\ddot{y}_c + C_0 y_c + C_x\alpha = 0 \\ J\ddot{\alpha} + C_x y_c + K\alpha = 0 \end{array}$$

where

$$C_0 = \sum C_i, \quad C_x = \sum C_i x_i, \quad K = \sum C_i x_i^2$$

free vibrations are of the form

$$y_c = A\sin p_T t, \quad \alpha = B\sin p_T t$$

After substitution in (21) we obtain the frequency equation

(22) $\Delta = \begin{vmatrix} -mP_T^2 + C_0 & C_x \\ C_x & -IP_T^2 + K \end{vmatrix} = 0$

$\Delta = mIP_T^4 - (mK + IC_0)P_T^2 + C_0K - C_x^2 = 0$

The solution of the biquadratic equation defines the values of the free vibration frequencies of the "rigid" elastically mounted beam. These are the free vibration frequencies of the elastically mounted diesel generator, which do not consider the elastic properties of the diesel generator - frame structure. These frequencies are obtained by applying existing calculation methods.

Forced oscillations are described by the system of differential equations (15), with F is the force that excites oscillations.

Multi-cylinder internal combustion engines have balanced inertial forces. Oscillations are excited by an imbalance of the generator rotor or because of inhomogeneity of the stator magnetic field. These forces, regardless of their different nature, are harmonic functions of the rotational frequency.

(23) $F = F_0 \sin \omega t$

Based on (23), the system of differential equations (15) takes the form:

$$M\ddot{\varphi} + C_1\varphi + C_2y_c + C_3\alpha = F_0I(x_f)\sin \omega t$$
(24) $$m\ddot{y}_c + C_2\varphi + C_0y_c + C_x\alpha = F_0 \sin \omega t$$
$$I\ddot{\alpha} + C_3\varphi + C_xy_c + K\alpha = F_0x_f \sin \omega t$$

where:

$$C_1 = C + \sum C_i\vartheta^2(x_i)$$
(25) $$C_2 = \sum C_i\vartheta(x_i)$$
$$C_3 = \sum C_i\vartheta(x_i)x_i$$

The solution of (24) is of the form

$$\varphi = A_\varphi \sin \omega t$$
$$y_c = A_y \sin \omega t$$
$$\alpha = A_\alpha \sin \omega t$$

The amplitudes of oscillations are obtained as a result of solving the algebraic system of equations:

(26) $$\begin{vmatrix} (C_1 - M\omega^2)A_\varphi + C_2A_y + C_3A_\alpha = F_0\vartheta(X_f) \\ (C_y - m\omega^2)A_y + C_2A_\varphi + C_x\alpha = F_0 \\ (K - J\omega^2)A_\alpha + C_3A_\varphi + C_xA_y = F_0X_f \end{vmatrix}$$

The bending vibrations of the elastically mounted beam with distributed parameters are described by the equation:

(27) $M\ddot{\varphi} + C_1\varphi = F_0\vartheta(X_f)\sin \omega t$

When the frequency of free bending vibrations is satisfied

$p = \omega$

resonant bending vibrations occur.

The vibrations of the aggregate as a rigid body are described by the equations

(28) $$\begin{vmatrix} m\ddot{y}_c + c_yy_c + C_x\alpha = F_0 \sin \omega t \\ J\ddot{\alpha} + C_xy_c + K\alpha = F_0X_f \sin \omega t \end{vmatrix}$$

When it is fulfilled

$p_T = \omega$

resonant vibrations occur on the "rigid" elastically mounted beam, which is used to model the aggregate. The intensity of the bending vibrations depends on the value of the shape of the bending vibrations $\vartheta(x_f)$ in the cross section where the force is applied F. If $\vartheta(x_f) = 0$, then regardless of the magnitude of the exciting force, it will not excite oscillations. At a large value of $\vartheta(x_f)$, a small force can excite substantial vibrations. Between bending vibrations and vibrations of the aggregate as a rigid body $(y_c(t), \alpha(t))$ there is only deformation connectivity. Unlike dynamic (inertial) connectivity, it is weak. That is why it is essential to consider partial subsystems:

- bending vibrations $\varphi(t)$
- oscillations like a rigid body $(y_c(t), \alpha(t))$.

and the ratio of the partial natural frequencies p and p_T with the excitation frequency ω.

The proximity of p to ω causes the excitation of intense bending vibrations, and the proximity of p_T to ω causes excitation of intense general oscillations.

Experimental results.

The object of the experimental research is a two-cylinder diesel generator with a power of kW at 720 rpm.

The vibrations were measured at 27 equally spaced points along the length of the frame.

The oscillation spectrum is dominated by a harmonic with a rotational frequency (720 rpm).

Point	RMS	point	RMS	point	RMS
1	14,5	10	12	19	6,6
2		11	11,8	20	6
3		12	10,5	21	5,6
4	15,6	13	10,5	22	5
5	16	14	9,8	23	4,6
6	15	15	8,8	24	4,25
7	15	16	8,6	25	4
8	15	17	7,6	26	4
9	13	18	7	27	3,8

The table presents data on the oscillations with a rotational frequency, and Fig. 4 presents a graphical representation of the law of variation of the fundamental harmonic along the length of the frame.

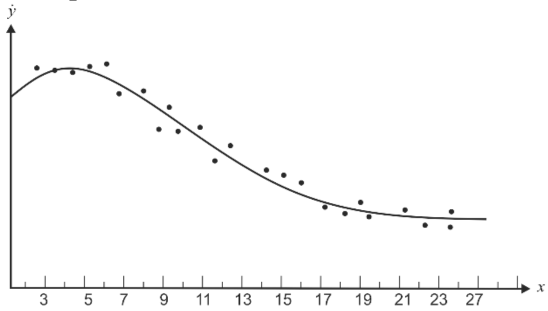

Fig.4. The law of variation of the fundamental harmonic along the length of the frame.

Fig. 5-a shows the experimentally obtained bending oscillations. The shape of the bending oscillations is visible.
Fig. 5-b shows the experimentally obtained total oscillations (oscillations as a rigid body). There is a large difference in the oscillations at both ends as a result of the uneven distribution of the mass.

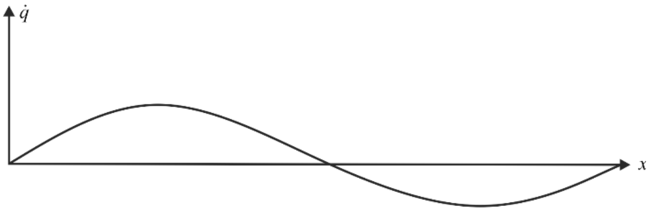

Fig.5-a. Experimentally obtained bending oscillations.

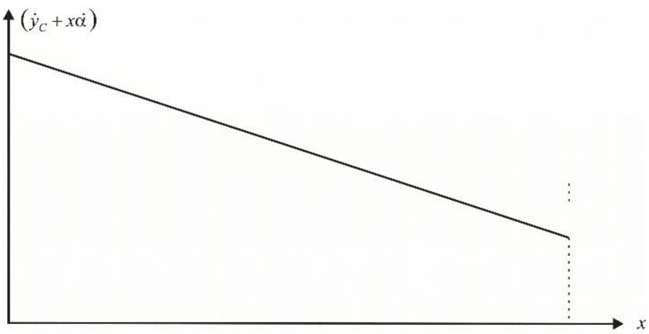

Fig.5-b. Experimentally obtained total oscillations.

Fig. 6 presents some spectra of the oscillations at several points of the frame.

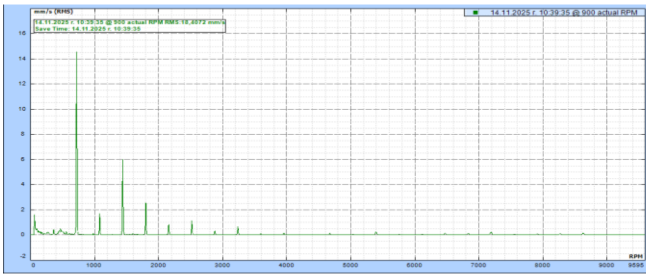

Fig.6-1. Spectra of the oscillations at several points of the frame, example-1.

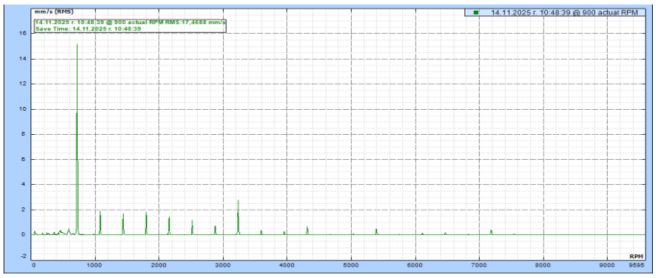

Fig.6-2. Spectra of the oscillations at several points of the frame, example-2.

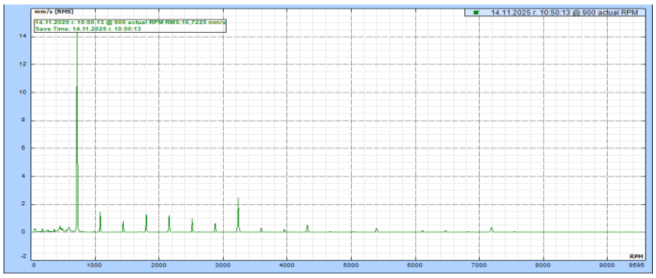

Fig.6-3. Spectra of the oscillations at several points of the frame, example-3.

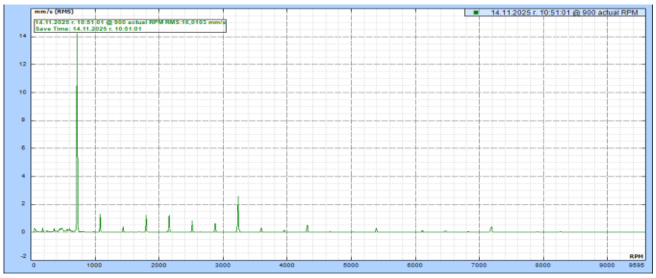

Fig.6-4. Spectra of the oscillations at several points of the frame, example-4.

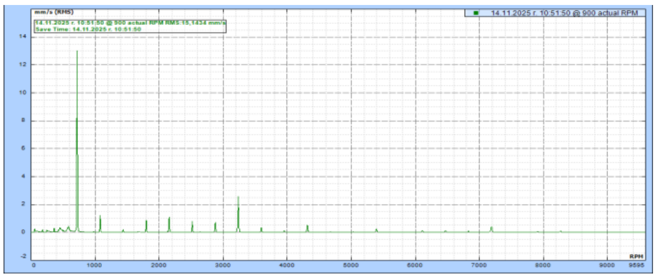

Fig.6-5. Spectra of the oscillations at several points of the frame, example-5.

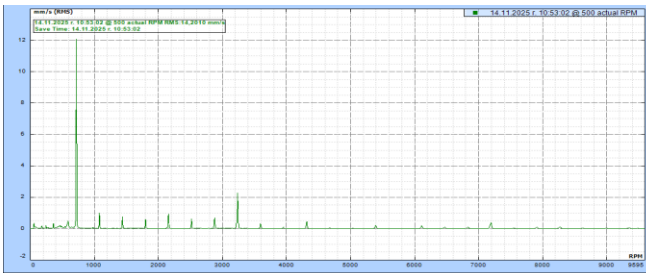

Fig.6-6. Spectra of the oscillations at several points of the frame, example-6

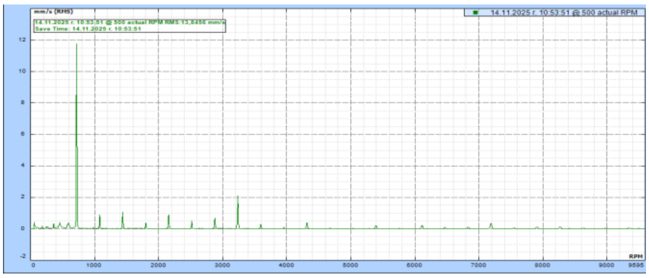

Fig.6-7. Spectra of the oscillations at several points of the frame, example-7.

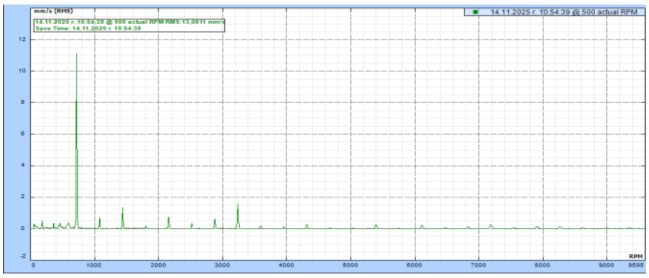

Fig.6-8. Spectra of the oscillations at several points of the frame, example-8.

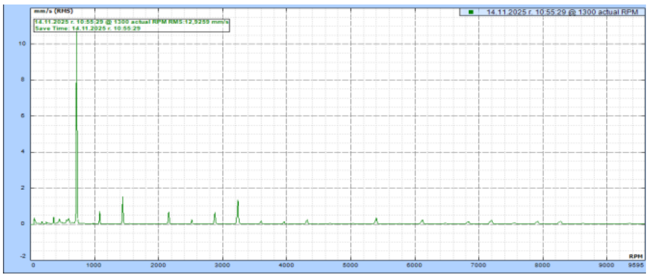

Fig.6-9. Spectra of the oscillations at several points of the frame, example-9.

CONSLUSION

The presented theoretical and experimental results determine the need to take into account the bending oscillations when studying the total oscillations of elastically mounted multi-cylinder diesel generators.

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