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Algorithm for Finding Optimal Placement of Objects in a Machining Section in the Presence of Alternative Technological Processes

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***Abstract*— In the context of digitalization of production and development of the Industry 4.0 concept, the optimization of production processes and the effective organization of production areas are gaining increasing importance. An essential task in the design of production systems is the optimal placement of the technological equipment in order to minimize the material flows, reduce transport distances and increase the efficiency of the production process. This task is further complicated in case of presence of alternative technological processes, allowing for different routes of parts processing.**

Based on a developed model, an optimization problem is formulated in the present paper, aimed at minimizing the transport distances and the time for parts transportation during the process of production. To solve the problem, an algorithm, based on heuristic methods for searching for optimal equipment placement is proposed. Computational experiments have been conducted, which illustrate the possibilities for reducing the intra-production transport, and thus for more even loading of the machines and shortening the production cycle.

The proposed approach corresponds to the principles of the Industry 4.0 concept and creates conditions for integrating intelligent planning and optimization methods into modern production systems.

Keywords*— *alternative technological processes; machining; material flows; production optimization; technological routes.

I. Introduction

Optimization problems for finding the optimal placement of objects in a production area are usually solved by means of the so-called flexible computational model, which specifies for each operation a list of equipment, on which it can be performed [1]-[10]. The model allows to use alternative technological processes for manufacturing a specific product and rests on the following assumptions:

- equipment can only be deployed on fixed sites with a known location;
- there is equality between the number of machines and the number of sites;
- the list of products, which must be manufactured in the particular production section is known, and their production schedule is also known;
- for each product, a technological route for its manufacturing is set, which is a list of the necessary operations to perform in order to have it ready;
- for each operation, a list of machines on which it can be performed, is specified;
- for each operation, the time for its execution is determined;
- each machine has its own time pool, which limits its working time, and hence the number of operations, which can be performed on it.

The flexible and the rigid computing models differ between each other in that the flexible one specifies for each operation a list of machines, on which it can be executed, while the rigid model can be regarded as just a special case of the flexible one, since it lists only one machine, on which the respective operation can be performed. What is more, the flexible method allows to use alternative technological processes to manufacture one and the same part, which means that the workpieces may undergo a different sequence of technological operations.

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II. MATERIALS AND METHODS

The task of finding the optimal placement of objects in a production area includes a number of stages, and one of them is to draw a graph of the technological route for manufacturing the specific product [11] – [13]. Another stage is the formulation of the task as an optimization problem and defining both its objective function and the model constraints, which must be imposed on the possible values of the edge flows of the graph [14]-[19]. These constraints can be divided into three groups [12]. Those, belonging to the first group, are linear equations, which form together a system of linear equations of the type:

$$\sum_{i=0}^{n+1} a_{k+1,l,i} = \sum_{j=0}^{n+1} a_{k,j,l} \,, \qquad (1)$$

where k is the operation number, or the level of the graph; n – number of machines in the production area; l – number of the machine, on which the k^{th} operation can be performed; j – number of the machine, on which the $(k-1)^{th}$ operation can be performed; i – number of the machine, on which the $(k+1)^{th}$ operation can be performed. These equations ensure equality of the input and output flows for each vertex of the technological graph.

The constraints from the second group also form a system of equations, having the form:

$$\sum_{i=0}^{n} a_{b,0,i} = n_b \,, \qquad (2)$$

where b is the number of the first operation of the technological process; n – number of machines in the production area; i – inventory number of the machine, on which the first operation of the technological process can be performed; n_b – number of products to be manufactured according to a given technological process (production schedule).The production schedule is used as an output parameter of the system, which means that the volume of work, which is set by the production schedule, is ensured.

According to the third type of constraints, the manufacturing routes must also satisfy the following system of inequalities in addition to the constraints from the first and second groups:

$$\sum_{b=1}^{m} \sum_{i=0}^{n+1} a_{b,l,i} \,.\, t_b < T_i \,, \qquad (3)$$

where b is the number of the operations, which can be assigned to the machine; n – number of units of equipment in the production area; m – number of operations to be performed in the production area; l – inventory number of the machine, to which an operation, preceding operation b, can be assigned; Ti – effective working time of the machine; t_b – required time to perform operation b. The third group of constraints is related to the time pool of the machines and the even load on the equipment.

In a number of problems from the graph theory, the parameter "throughput" of the edges of a graph is considered. In this case, the time pool of the equipment plays the same role as the throughput does in other tasks, e.g., for optimizing transport flows in a city. The difference is that these constraints are written not for the edges of the graph, but for its vertices. Moreover, each vertex is part of not one, but of several technological graphs, since, under the conditions of small-scale production, each machine takes part in processing many different products.

After setting the constraints on the values of the parameters $a_{i,j,k}$, which limit the possible options for production routes, it is necessary to express also the objective function as a quantity, depending on the variable parameters of the model, i.e., on the edge flows of the graph and the location of the machines.

Increase in the flow capacity is observed when the workpieces are moved along the edges of the graph, i.e. along the production route. Load flow capacity value is equal to the product of two numbers - the length of the route and the mass of the workpieces. The product of these numbers can be regarded as the price for transporting the material load along the corresponding edge of the technological route graph. Then, the objective function can be written as a linear combination of the flows along the edges:

$$C = \sum_{e=1}^{m} \sum_{f=0}^{n+1} \sum_{z=0}^{n+1} a_{e,f,z} \,.\, R_{f,z} \,.\, M_e \,, \qquad (4)$$

where e is the graph level, i.e. operation number; m – total number of the operations, performed in the production area; f, z – equipment inventory numbers; n – number of machines; $R_{f,z}$ – distance between the pairs of machines f and z; M_e – mass of the parts after the execution of the operation.

If the machine placement option is specified, then the value of $R_{f,z}$ is not a variable, but a constant parameter of the problem and M_e is also constant. Consequently, the only variable parameters are the edge flows of the technological route graph $a_{e,f,z}$. Thence, the objective function is a linear function of the variables $a_{e,f,z}$. The constraints, belonging to the first and second groups, are also linear, and together they form a system of undetermined linear equations. The system is indeterminate because the number of variables is greater than the number of equations in it, which means that it has an infinite number of solutions or that it cannot be solved in the standard way. And, the third group of constraints is a set of linear inequalities.

It can be concluded from the above that the formulated optimization problem is a problem of linear optimization (programming) [20]-[24]. Linear optimization is a method for finding the best solution (max or min) of an objective function under linear constraints (equalities or inequalities). An efficient algorithm for solving problems of this type, known as the simplex method, was proposed by George Danzig in 1949 [25], [26]. The simplex method, with its various modifications, is still used up to the present for optimization of linear objective functions.

By using the simplex method, it is possible to determine the optimal distribution of operations between technological equipment. The value of the load flow capacity, corresponding to this distribution, and the considered machine placement option, can also be determined. The determined value of the load flow capacity can be used to assess the quality of one or another planning decision, i.e., it can be used as an objective function for finding optimal equipment placement.

In the problem of finding the best option for placing technological equipment, the variable (equipment location) is a permutation, which means that there is a discrete optimization problem. For example, in the classic for this field Traveling Salesman Problem, the variable parameter (the planned route) can also be represented in the form of a permutation. Since the quadratic assignment problem, which is NP-hard [27], [28], is only a special case of The Traveling Salesman Problem [29], [30], the problem of finding an optimal planning decision is also an NP-hard one [27], [31]. Therefore, it is necessary to look for heuristic algorithms for its solution, which are only approximate and not exact .

One of the most efficient and at the same time simple to implement among the heuristic algorithms is the Simulated Annealing (SA) method [32], [34].

SA is a metaheuristic probabilistic global optimization algorithm, inspired by the metallurgical process of annealing metals, which involves heating and slow cooling. At a very low cooling rate of the metal, the atoms arrange themselves in a crystal lattice in such a way, that their energy is minimal. This leads to a reduction in the internal stresses and reaching a more stable state. SA finds good approximate solutions to complex combinatorial problems, allowing temporary acceptance of worse solutions to avoid local minima [32], [35].

The initial temperature (to which the metal is heated during the annealing process) and the cooling rate must be set as algorithm parameters. The rate of cooling is determined by the number of iterations. SA gradually reduces the temperature after each iteration. The pace of the temperature decrease is equal to the ratio of the initial temperature to the number of iterations. The algorithm stops working at the moment when the temperature drops to zero.

Initially, SA randomly finds an arbitrary point, which corresponds to one of the possible equipment placement options. Then, the value of the objective function for this point is determined. For this purpose, it is necessary to solve the linear programming problem for the considered machine placement option and to determine the possible load flow capacity. The result of the first iteration is automatically taken as a base (the best solution found so far), and therefore the resulting arrangement of the machines and the value of the objective function are accepted and saved as the current optimal solution.

Then, SA searches for a different equipment placement option by randomly moving to one of the points, located near the first point in the solution area. To do this, the algorithm swaps the places of several machines. At each new point, the objective function is calculated and compared with the best solution obtained to that moment. If the new solution is closer to the optimum, it is accepted as the best for the moment, and the search for new points continues .

Each iteration generates a new solution in the range of the base one. The value of the objective function is calculated for the new solution and compared to the value of the objective function for the best solution found to the moment. If the new solution is better, then it is accepted as a base solution with a certain probability, which depends the temperature and the difference in the values of the objective function.

It is possible for SA to return to a given point even though the best solution has not been found there. This possibility depends on the temperature and on the difference between the calculated objective function and the currently optimal objective function. The higher the temperature, the greater the probability of moving to another point.

The state of a thermodynamic system of a large number of particles, which is in equilibrium with its surroundings can be described by the Gibbs distribution [35]. When applied to the objective function optimization problem, this distribution looks in the following way:

$$P(x) = \begin{cases} e^{\left(\frac{OF(x_{bp}) - OF(x)}{T}\right)}, & \text{at } OF(x) \geq OF(x_{bp}), \\ 1, & \text{at } OF(x) < OF(x_{bp}) \end{cases} \tag{5}$$

where x is the permutation, for which the transition chance is determined; x_{bp} – "base" point; T – temperature for the given iteration; P(x) – chance for the permutation x to become a "base" one; OF(x) – value of the objective function x, calculated when solving the linear programming problem.

The gradual decrease in the temperature after each iteration leads to a decrease in the probability for the currently studied option for machine placement to be accepted as a base solution. Allowing a transition to a solution with a less favorable objective function value allows the algorithm to overcome local optima and increases the probability of reaching to the global optimum.

The number of iterations can be set before starting the algorithm. The more iterations, the more time for calculations will be needed, but more permutations will be considered and the solution will be closer to the optimal one.

It should be noted that all linear programming problems, solved at the different iterations, using the simulated annealing method, have a number of similar features. On the one hand, the set of variables is the same in all problems. The same is also the search criterion (minimum). All constraints can be divided into three groups. The first group of constraints is determined by the technological routes and does not change with the different options for equipment placement, so they stay the same for each iteration. The second group of constraints is determined by the production schedule of the products and thus it also does not depend on the machine placement option, i.e., these constraints remain constant for all iterations as well. Finally, the third group of constraints is determined by the equipment working time pool, which is also constant for all iterations. This means that in all problems, solved by linear programming, the constraints on the variable parameters completely coincide, i.e. the ranges of permissible values are identical.

The only difference between these problems can be seen in the coefficients $R_{f,z}$ and M_e in the objective function. M_e determines the mass of the workpieces, and $R_{f,z}$ – the distance between the machines, i.e., equipment location.

Ultimately, all linear programming problems, solved by the considered algorithm, have the same range of permissible values and a different objective function.

Based on the exposed above, an algorithm can be developed to find the optimal placement of machines in a production area. This algorithm uses the simulated annealing method to explore possible equipment placement options. To assess the quality of each option, a linear programming problem is formulated, which is solved using the simplex method. The algorithm is composed of the following steps:

Step 1. Start (start of the algorithm).

Step 2. Entering the following source data:

- distances between the sites for equipment placement;
- technological routes;
- mass of the workpieces to be moved;
- production schedule;
- initial temperature T;
- cooling pace dT;
- number of iterations, etc.

The output parameters of the model are formulated by means of them.

Step 3. Defining the parameters:

- P_{base} – base placement;
- $P_{current}$ – current placement;
- H_{base} – base value of the objective function;
- $H_{current}$– current value of the objective function;
- dT – temperature decrease step.

Step 4. Compiling a matrix of constraints on the values of the edge flows $a_{i,j,k}$.

Step 5. Compiling a random starting equipment placement option P.

Step 6. Compiling a matrix of the distances between the machines $R_{f,z}$.

Step 7. Calculating the coefficients of the linear function C.

Step 8. Finding min H(P) of function C by solving the linear programming problem.

Step 9. Checking whether $H(P) < P_{current}$.

If $H(P) < P_{current.}$, then the following is assumed:

$P_{current} = H(P)$; $P_{base} = H(P)$; $P_{current.} = P$; $P_{base} = P$.

Proceeding to step 13 then.

If $H(P) \geq P_{current}$, then step 10 follows.

Step 10. Calculating the probability:

$B(P) = e^{(H_{base} - H(P))/T}$.

Step 11. Choosing a random number x, so that:

$0 < x < 1$.

Step 12. Checking if $x < B(P)$.

If $x < B(P)$, it is assumed that

$P_{base} = H(P)$; $P_{base} = P$.

If $x \geq B(P)$, step 13 follows.

Step 13. Lowering the temperature: T = T – dT.

Step 14. Checking whether T > 0.

If T > 0, random placement of the equipment near P_{base} is arranged. And then the algorithm returns to step 6.

If T ≤ 0, then step 15 follows.

Step 15. The resulting equipment placement option $P_{current}$ is accepted as optimal.

Step 16. End.

Based on the proposed algorithm, a software package has been developed, which allows for the selection of alternative technological processes by which products can be manufactured in a production area.

III. RESULTS AND DISCUSSION

The following example is considered to demonstrate the operation of the developed software. Sixteen (16) machines are to be placed in a manufacturing section. The number of sites on which the machines will be located is also 16. Table 1 presents a list of the machines with assigned unique inventory numbers (from 1 to 16).

Eleven (11) different products must be made, labeled with letters from A to K. For each product, the following is set:

- technological process for its manufacture;
- production schedule;
- mass of the workpieces;
- list of machines, on which each operation can be performed;

- time for performing each operation.

TABLE 1. LIST OF MACHINES IN A MACHINING SECTION

N	Model	Type
1	DMU 65 mono BLOCK	5 Axis Machining Center
2	NEF 600	CNC Lathe Machine
3		
4		
5		
6	CTX beta 800 TC	Turn & Mill Complete Machining Center
7		
8	QV 127	Vertical CNC Machining Center
9		
10	WHN 130Q	Milling and boring machine
11	Vertical Mate 35	CNC Vertical Multi-Process Grinding Machine
12	OMAXMicroMax	Abrasive Waterjet Machine
13	Cybertech SLi-30FT	CNC Wire Cut Machine
14	NHX	High-Speed Horizontal Machining Center
15	Gecam G6 WET	Deburring Machine
16	CTV 160	Vertical pick-up lathes

The operating schedule of the machining section is also known, as well as the positions, where the technological equipment can be located (Fig. 1).

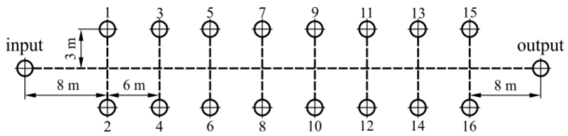


Fig.1. Schedule of the production section with positions for placing the technological equipment

Parts A, B, C and D can be manufactured using several alternative technological processes. Parts A, C, and D, in accordance with the available machines, can be manufactured, following three alternative technological processes, and part B - using four alternative processes. Parts from E to K can be manufactured by means of only one possible technological process.

Each operation is also assigned a unique identification number. Each technological route begins with an arbitrary first operation "workpiece delivery", without specifying the machine, on which it will be performed, or the duration of the operation.

Part “A” (Fig.2 and Fig.3), considered as an example, can be manufactured using three alternative technological processes.

Here is what is also known regarding part"A": the production schedule n = 118 pcs.; the mass of the workpieces M_e = 15.36 kg.

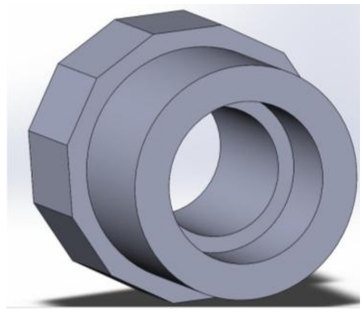


Fig.2. 3D model of part “A”

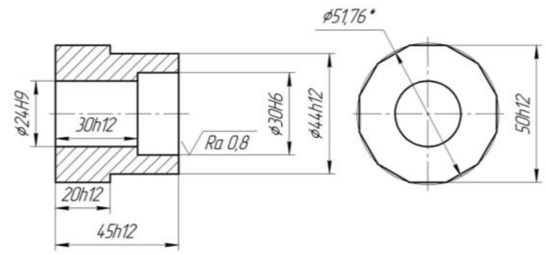


Fig.3. Drawing of part “A”

The remaining data, needed for carrying out the calculations, are presented in Table 2.

TABLE 2. OPERATIONS FOR MANUFACTURING PRODUCT “A”

Product	Technological route	Input operations	Duration of the operation, min	List of machines
A1	1→2→3→4	1	-	-
		2	12,46	2, 3, 4
		3	7,22	8, 9
		4	9,64	11
A2	5→6→7	5	-	-
		6	29,76	6, 7
		7	9,64	11
A3	8→9→10	8	-	-
		9	15,64	8, 9
		10	16,25	5

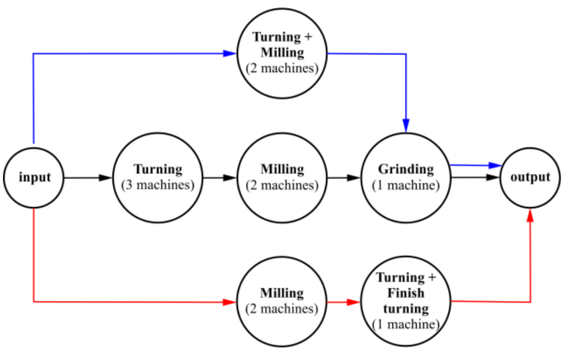
It is first necessary to calculate in advance the distances between the positions shown in Figure 1. The paths, along which the workpieces can be moved, are indicated by a dashed line. It is convenient in this case to use the Manhattan metric (rectangular metric, L_1 norm) in order to determine the distances. Workpiece transportation can be considered as a sequential movement along horizontal and vertical sections, corresponding to the real directions of the transport routes. The distance between two positions in the grid space is calculated as the sum of the absolute differences of their coordinates. The use of a rectangular metric is justified due to the configuration of the section and the limitations of the intra-factory transport, in which diagonal movement is practically not possible. The obtained results are presented in Table 3 (0 – input, 1 ÷ 16 – technological equipment, 17 – output).

TABLE 3. LENGTH OF THE TRANSPORT DISTANCES BETWEEN POSITIONS IN A MACHINING SECTION

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
0	0	11	11	17	17	23	23	29	29	35	35	41	41	47	47	53	53	53
1	11	0	6	12	12	18	18	24	24	30	30	36	36	42	42	48	48	48
2	11	6	0	12	12	18	18	24	24	30	30	36	36	42	42	48	48	48
3	17	12	12	0	6	12	12	18	18	24	24	30	30	36	36	42	42	42
4	17	12	12	6	0	12	12	18	18	24	24	30	30	36	36	42	42	42
5	23	18	18	12	12	0	6	12	12	18	18	24	24	30	30	36	36	36
6	23	18	18	12	12	6	0	12	12	18	18	24	24	30	30	36	36	36
7	29	24	24	18	18	12	12	0	6	12	12	18	18	24	24	30	30	30
8	29	24	24	18	18	12	12	6	0	12	12	18	18	24	24	30	30	30
9	35	30	30	24	24	18	18	12	12	0	6	12	12	18	18	24	24	24
10	35	30	30	24	24	18	18	12	12	6	0	12	12	18	18	24	24	24
11	41	36	36	30	30	24	24	18	18	12	12	0	6	12	12	18	18	18
12	41	36	36	30	30	24	24	18	18	12	12	6	0	12	12	18	18	18
13	48	42	42	36	36	30	30	24	24	18	18	12	12	0	6	12	12	12
14	48	42	42	36	36	30	30	24	24	18	18	12	12	3	0	12	12	12
15	53	48	48	42	42	36	36	30	30	24	24	18	18	12	12	0	6	6
16	53	48	48	42	42	36	36	30	30	24	24	18	18	12	12	6	0	6
17	53	48	48	42	42	36	36	30	30	24	24	18	18	12	12	6	6	0

All information, required for the material flow optimization, is entered into the specialized software.

Both turning and milling operations are necessary to be performed for manufacturing part “A”. The opening Ø30H6 has high roughness requirements and therefore, either grinding or fine turning is necessary to apply for its finishing. Three alternative technological processes have been compiled and presented in the form of a graph in Fig.4.



ATP1: input→Turning→Milling→Grinding→output
ATP2: input→Turning+Milling→grinding→output
ATP3: input→Milling→Turning+Finish turning→output

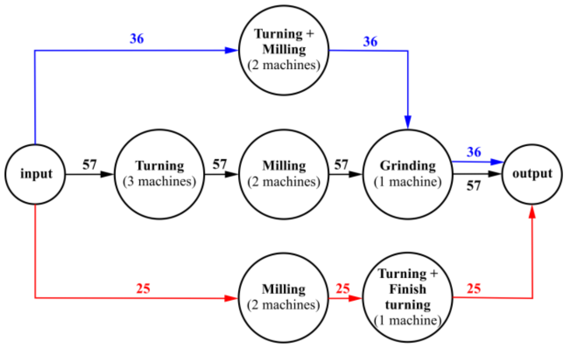
Fig.4. Alternative technological processes for manufacturing product “A”

The first technological process (ATP1) includes initial turning, followed by milling, and finally applying grinding as a finishing process.

The second alternative technological process (ATP2) provides for combining turning and milling operations on a turning-milling machining center. The accuracy characteristics of the machining centers, available in the section, do not allow for fine turning, and therefore the workpiece must be subjected to a finishing operation - grinding.

The third alternative technological process (ATP3) begins with milling, after which the workpiece is transferred for turning to a machine, whose accuracy characteristics allow for fine turning (instead of grinding).

As a result of the performed calculations, the following distribution of work between the competing ATPs has been proposed (Fig.5).



ATP1: input→Turning→Milling→Grinding→output (57 units)
ATP2: input→Turning +Milling→Grinding→output (36 units)
ATP3: input→Milling→Turning+Finish turning →output (25 units)

Fig.5. Technological graph at an operations level - work distribution between the alternative technological processes

57 parts are to be manufactured, following ATP1, 36 parts by means of ATP2, and 25 - using ATP3.

The graph, presented in Fig. 5, is in a simplified form, only presenting the operations. However, the technological graph must be constructed for the specific units of the technological equipment.

At the equipment level, the same graph will look like the one, presented in Fig. 6.

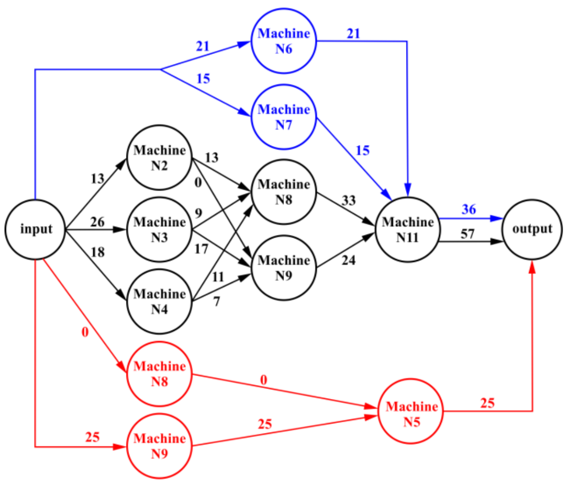


Fig.6. Technological graph at the equipment level with calculated edge flows

When calculating production routes, the developed software does not select just one of the alternative technological processes, but distributes the entire workload between all of them. On the one hand, such an organization of production increases its technological flexibility. On the other hand, only with such a formulation of the material flow optimization problem, can it be reduced to a linear programming problem.

The calculations are not performed for each individual part and each graph separately, but simultaneously for all manufactured products and technological graphs, since the total load on the equipment in the section is taken into account. As a result, it is possible to ensure an even load on the machines in the section, avoid downtime, and increase the overall productivity of the manufacturing system.

IV. Conclusions

Based on a developed mathematical model, the problem of optimizing material flows in a production system has been formulated. For solving this problem, an algorithm is proposed, developed by combining linear programming methods and the method of simulated annealing. The algorithm takes into account the distance between the positions where the machines are located, production schedule, mass of the workpieces, technological routes, working time pool of the machines and duration of the operations. The algorithm allows to find both an option for optimal equipment placement and optimal production routes.

For implementing the algorithm, a software package has been developed, which can solve the optimization problem for a production area, described by a flexible computational model.The application of the software allows to find an option for optimal equipment placement, as well as to draw up optimal production routes.

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