

State Controller Synthesis for an Electrohydraulic Power Transmission System

Hristo Nedev Hristov

Dept. Power Engineering, Technical University of Gabrovo
Center of Competence "Smart Mechatronic, Eco-and Energy-Saving Systems and Technologies"
Gabrovo, Bulgaria
christo@tugab.bg

Abstract— This paper deals with the synthesis of state controller for an electrohydraulic power transmission system. There are shown the mathematical model of the electrohydraulic system and state controller synthesis method with state space feedbacks. The controller coefficients are obtained from preliminary defined error performance index. The response curves are obtained from computer simulation. The results are given in a few graphics.

Keywords— *electrohydraulic transmission systems, state controller, modeling and simulation*

I. INTRODUCTION

Hydraulic power transmission systems, in comparison with other types of drives, have lower stiffness and damping, which is a disadvantage in conditions of significant dynamic loads [1], [2], [3], [4]. The control elements of aircraft, presses and mobile machines are inertial objects [5], [6] with a significant mass, are mounted on elastic supports and are connected to the drive system with elastic mechanical connections.

The design and synthesis of stabilizing and correcting devices is an important stage in the design of hydraulic systems. One of the approaches to increasing the damping of the system is the introduction of a state regulator [2], [3]. The advantage of the state regulator is primarily that it provides high dynamic characteristics of the system in combination with increased damping. Thanks to the state regulator, the system responds faster to external influences. Popular approaches include classical feedback and compensation schemes, as well as variants of state observers and Kalman filters, including linear and nonlinear extensions [7], [8], [9], [10]. Analogous control approaches are also used in electropneumatic positioning systems [11], [12]. When analyzing the performance of electro-hydraulic traction systems, it is important to evaluate positioning errors in dynamic mode [13], [14].

II. STATE CONTROLLER DESIGN

The state controller is multi-loop, in which the state parameters are covered by feedback loops. The transfer function of the open electrohydraulic system consists of the product:

$$W(s) = W_{\text{exy}}(s) W_x(s) W_m(s) \frac{1}{T_y(s)} \quad (1)$$

where $W_{\text{exy}}(s)$ - transfer function of electrohydraulic amplifier, often approximated to a oscillating unit; $W_x(s)$, $W_m(s)$ - transfer function of the hydraulic and mechanical part of the system (can be approximated to aperiodic units of the first order); $\frac{1}{T_y(s)}$ - an integrating

unit converting the velocity coordinate into the position coordinate of the system. In this way, the typical electrohydraulic system is described by a 5th order differential equation. If in the system the speed of the electro-hydraulic amplifier is significantly greater than the speed of the hydraulic cylinder together with the load ($\omega_{\text{exy}} \gg \omega_o$), then without significant error it can be considered that the system can be approximated to a third-order system and therefore it is sufficient to introduce feedbacks on three parameters – displacement, speed, acceleration or pressure drop in the cylinder. steam distributor, which has a relatively small natural frequency.

The meaning of feedback on the state parameters is concluded in the following. The transfer function of the closed loop consists of a section of the control system $W(s)$ covered by feedback (with transfer function ($W_{fb}(s)$)):

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$$\Phi(s) = \frac{W(s)}{1 + W(s)W_{fb}(s)} \quad (2)$$

$$\text{If } W(s)W_{fb}(s) \gg 1, \text{ hence } \Phi(s) = \frac{1}{W_{fb}(s)}. \quad (3)$$

Thus, in the frequency range in which the inequality $W(s)W_{fb}(s) \gg 1$ is satisfied, the characteristic of the loop is generally determined by the characteristics of the feedback.

To implement the system with state control, it is necessary to install speed and acceleration sensors or double differentiation of the signal from the position sensor. When installing the sensors, the cost of the system increases, and complexity arises in their installation. In the case of signal differentiation, the system becomes very sensitive to high-frequency impacts and obstacles. An alternative solution is the use of a “state observer” or systems with state control with an “observer”. The main task in improving the dynamics of hydraulic drive systems is to eliminate the resonance of the output part of the drive, while increasing the speed gain coefficient K_{vy} , which determines the system’s speed and static stiffness.

One of the directions of the theory of systems is connected with design of regulators providing the desired dynamics of the system, based on creation of feedback on the coordinates of the state, transmitting to closed-position tracking systems the desired arrangement of roots of the characteristic equation of its linear model (poles of the transfer function). The arrangement of roots of characteristic equation of the drive system on the complex plane in many cases completely determines the regulated indicators of the dynamics of the system and has visual interpretations [3]. A structural diagram of an electrohydraulic drive system with a state regulator is shown in Fig. 1.

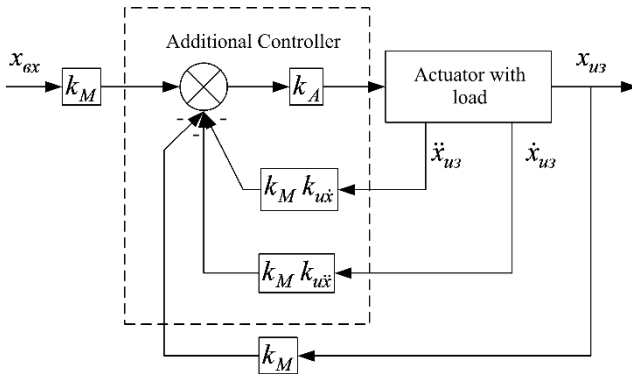


Fig. 1. Hydraulic system with state controller.

The transfer function of the linear model of the system with this state regulator, reflecting its most essential dynamic properties, has the form:

$$\frac{x_{u3}}{x_{6x}} = \frac{1}{\frac{s^3}{\omega_0^2 K_{vy}} + \left[\frac{2\xi_0}{\omega_0 K_{vy}} + \bar{k}_{u3} \right] s^2 + \left[\frac{1}{K_{vy}} + \bar{k}_{u3} \right] s + 1} \quad (4)$$

where:

$$k_{u3} = \frac{k_{u3}}{k_M}; \quad \bar{k}_{u3} = \frac{k_{u3}}{k_M}.$$

For $s = \frac{s}{\omega_c}$ (where: $\omega_c = \sqrt[3]{\omega_0^2 K_{vy}}$) the characteristic equation is:

$$s^{-3} + as^{-2} + bs^{-1} + 1 = 0, \quad (5)$$

where a and b are:

$$a = \left[\frac{2\xi_0}{\omega_0 K_{vy}} + \bar{k}_{u3} \right] \sqrt[3]{\left(\omega_0^2 K_{vy} \right)^2}; \quad (6)$$

$$b = \left[\frac{1}{K_{vy}} + \bar{k}_{u3} \right] \sqrt[3]{\omega_0^2 K_{vy}} \quad (7)$$

For each set of design parameters of the system $\{k_i\}$, there corresponds a pair of generalized coefficients $a(k_i), b(k_i)$, defining a mapping point on the coefficient plane a, b .

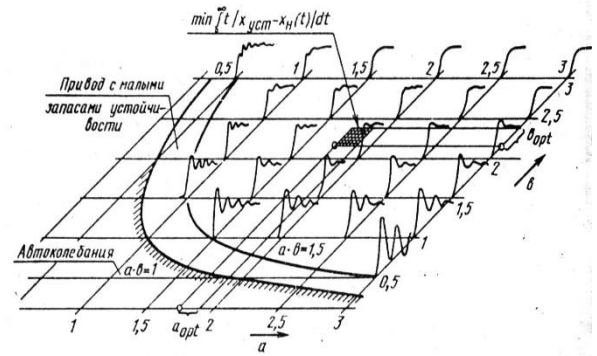


Fig. 2. System transients with a step input signal for different points on the generalized coefficient plane (a, b) - [1, 3].

The stability limit is determined by the ratios $a > 0, b > 0, a.b = 1$. A region with a margin of stability in amplitude, above 0.33, is located above the curve $a.b = 1.5$ (Fig. 2).

Fig. 2 [1, 3] shows an example of the transient processes of the system with a step input signal for different points on the plane of the generalized coefficients (a, b) . As practice shows, the best compromise between the fast action of the mechanism and its oscillation is achieved by minimizing an integral criterion characterizing the magnitude of the error between the desired trajectory of the output link and the duration of the transient process [1], [2], [3]:

$$ITAE = \int_0^{\infty} \frac{t}{E(t)} dt, \quad (8)$$

where, $E(t)$ - error;

$$E(t) = \frac{1 - x_{u3}(t)}{x_{ycm}} \quad (9)$$

The optimal synthesis task in the sense of minimum of the integral criterion determines a set of controller parameters $k_{onm} = \{K_{vy}; r_e; \bar{k}_{u\dot{x}}; \bar{k}_{u\ddot{x}}\}_{onm}$, such that the integral criterion k_{onm} takes on a minimum value, i.e. $ITAE(k_{onm}) = \min ITAE$.

Research shows [2] that the optimal values $a_{onm}; b_{onm}$ of the parameters that provide a minimum of the criterion $ITAE$ have the following estimates:

$$a_{onm} = 1,75 - 1,89; \quad (10)$$

$$b_{onm} = 2,15 - 2,38. \quad (11)$$

Using these estimates, it can be obtained that the optimal ratio between the natural frequency of the drive system ω_0 and the speed gain K_{vy} (without additional correcting devices) is in the range:

$$\left[\frac{K_{vy}}{\omega_0} \right]_{onm} = 0,27 - 0,35, \quad (12)$$

However, at these ratios undesirable oscillatory processes may occur in the system, which is unacceptable in some operating modes.

To obtain the desired dynamics of the system with a fast-damping transient process and satisfactory frequency properties, it is necessary to determine such $\bar{k}_{u\dot{x}}$ and $\bar{k}_{u\ddot{x}}$, which would ensure that the imaging point falls into a region corresponding to optimal values in terms of speed a_{onm} and b_{onm} . The values of these coefficients from (6) and (7) are obtained in the following form:

$$\left[\bar{k}_{u\dot{x}} \right]_{onm} = \frac{b_{onm}}{\sqrt[3]{\omega_0^2 K_{vy}}} - \frac{1}{K_{vy}} \quad (13)$$

$$\left[\bar{k}_{u\ddot{x}} \right]_{onm} = \frac{a_{onm}}{\sqrt[3]{(\omega_0^2 K_{vy})^2}} - \frac{2\xi_0}{\omega_0 K_{vy}} \quad (14)$$

The determination of the main parameters of the electrohydraulic system occurs after identification of the parameters made on a laboratory stand and a corresponding identification procedure [7].

The results obtained from the experimental studies and identification [7] are:

Natural frequency of the electrohydraulic system with a spring load with a constant $C_T = 51000$ N/m - $\omega_0 = 20.19$,

Damping coefficient $\xi_0 = 0.921$,

Speed gain coefficient $K_{vy} = 2.43$ s⁻¹

The following values are accepted for the coefficients:

$$a = 1.75; b = 2.3;$$

From equations (13) and (14) we obtain:

$$\bar{k}_{u\dot{x}} = 207.53; \quad \bar{k}_{u\ddot{x}} = 93.62; \quad (15)$$

III. MATHEMATICAL MODEL OF AN ELECTRO-HYDRAULIC SYSTEM WITH A STATE REGULATOR

The mathematical model of the electrohydraulic system is shown in [7, 8]. To this model are added the equations of the additional feedback on the state variables:

Equation of the hydraulic cylinder with the load:

$$m \frac{d^2 y}{dt^2} + f \frac{dy}{dt} + C_S y = p_1 A_1 - p_2 A_2 - F_C(v) \quad (16)$$

where: $F_C(v) = F_{C0} \text{Sign}(v)$ и $v = \frac{dy}{dt}$

- Pressure in the cylinder chambers

for $x_a < x < x_m$

$$\frac{dp_1}{dt} = \frac{E}{V_1(y)} \left[\frac{G(x) \sqrt{p_s - p_1}}{\bar{Q}_1} - A_1 \frac{dy}{dt} \right]$$

$$\frac{dp_2}{dt} = \frac{E}{V_2(y)} \left[-\frac{G(x) \sqrt{p_2 - p_0}}{\bar{Q}_2} + A_2 \frac{dy}{dt} \right] \quad (17)$$

for $-x_a < x < -x_m$

$$\frac{dp_1}{dt} = \frac{E}{V_1(y)} \left[\frac{G(x) \sqrt{p_1 - p_0}}{\bar{Q}_1} - A_1 \frac{dy}{dt} \right]$$

$$\frac{dp_2}{dt} = \frac{E}{V_2(y)} \left[-\frac{G(x) \sqrt{p_s - p_2}}{\bar{Q}_2} + A_2 \frac{dy}{dt} \right]$$

(18)

- Hydraulic conductivity:

$$G(x) = \begin{cases} K_o[x - x_a \text{Sign}(x)] & x_a < |x| < x_m \\ G_m \text{Sign}(x) & |x| = x_m \end{cases} \quad (19)$$

$$K_o = \mu b \sqrt{\frac{2}{\rho}} \quad G_m = \mu b (x_m - x_a) \sqrt{\frac{2}{\rho}}$$

- Volumes of the left and right cylinder chambers

$$V_1(y) = V_{01} + A_1 y \quad V_2(y) = V_{02} + A_2 y \quad (20)$$

- Physical limitations in the hydraulic system:

$$|y| \leq y_m \quad |v| \leq v_m \quad |x| \leq x_m \quad (21)$$

$$0 < p_1 < p_S \quad 0 < p_2 < p_S \quad (22)$$

- Servo valve baffle rotation:

$$T_f^2 \frac{d^2 \varphi}{dt^2} + 2\zeta_f T_f \frac{d\varphi}{dt} + \varphi = k_i i - k_{rx} x \quad (23)$$

- Displacement of the servo valve plunger

$$T_{\varphi x} \frac{dx}{dt} + x = k_{\varphi x} \varphi \quad (24)$$

Electromechanical converter

$$T_i \frac{di}{dt} + i = k_U U_P \quad (25)$$

- Electronic amplifier and feedback on state variables –

$$y, \quad \frac{dy}{dt} = v, \quad \frac{d^2 y}{dt^2} = a$$

$$U_P = K_a \Delta U \quad \Delta U = U_v - U_y - U_{\dot{y}} - U_{\ddot{y}}$$

$$U_y = K_y y \quad U_{\dot{y}} = K_{\dot{y}} \frac{dy}{dt} \quad U_{\ddot{y}} = K_{\ddot{y}} \frac{d^2 y}{dt^2} \quad (26)$$

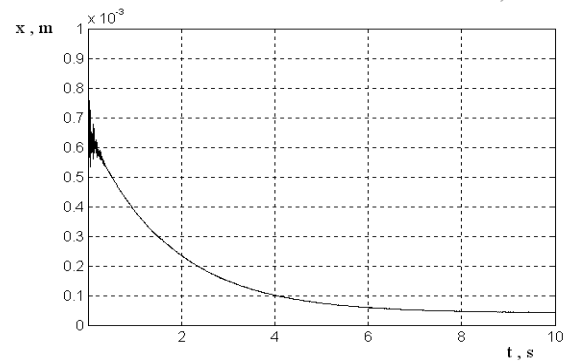
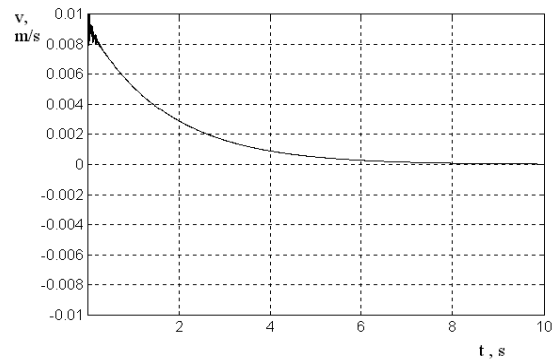
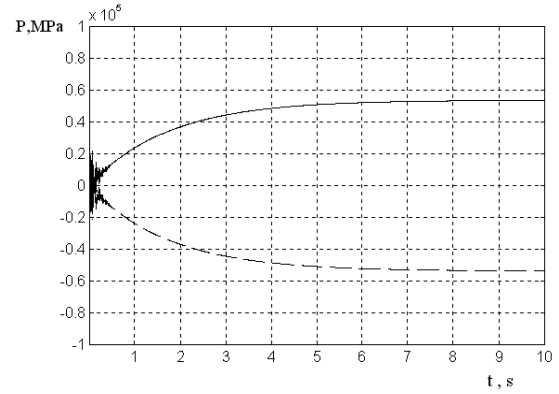
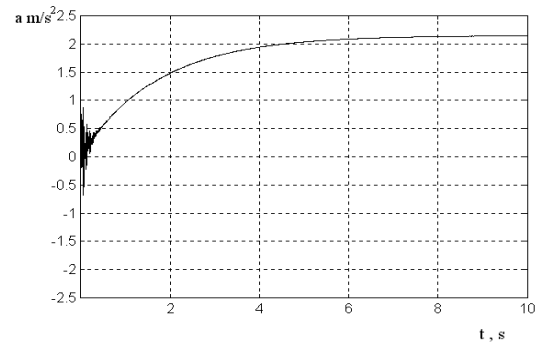
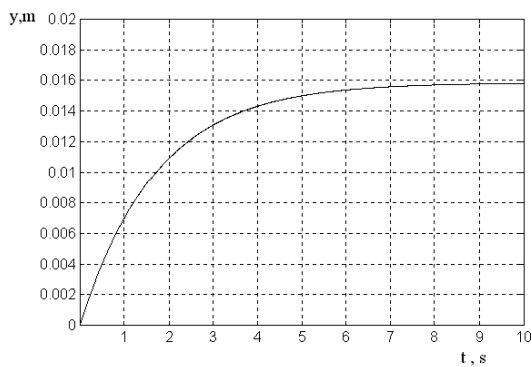


Fig. 3. Step responses in the no-load system

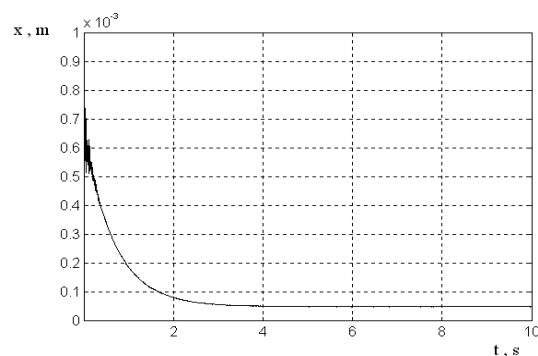
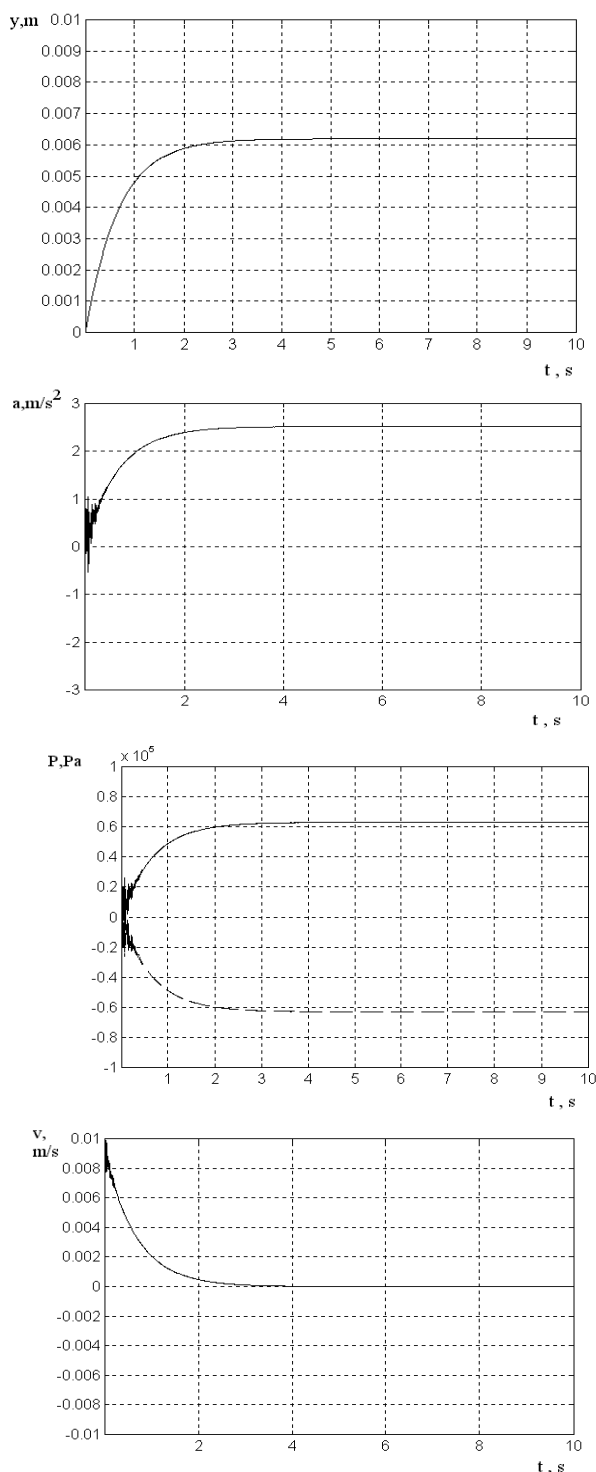


Fig. 4. Step responses in the system with load ($C_T = 51000 \text{ N/m}$)

IV. RESULTS FROM COMPUTER SIMULATION OF THE MATHEMATICAL MODEL

Equations (17-27) are used in the computer modeling of the electrohydraulic drive system with a state regulator. The dynamic processes obtained after the simulation of the mathematical model are shown in Fig. 3 and 4. The numerical values of the coefficients used in the simulation of the model and their determination are shown in [7], [8]. Fig. 3 shows the transient processes in the electrohydraulic system with a state regulator, without load elasticity - $C_T = 0 \text{ N/m}$. Fig. 4 shows the transient processes in the electrohydraulic system with a state regulator, with load elasticity load $C_T = 51000 \text{ N/m}$.

V. RESULTS AND CONCLUSION

From the analysis of the transients obtained after the computer simulation, it can be concluded:

1. The system speed increases, while the type of transient remains aperiodic.
2. Due to the feedback from the acceleration, high-frequency oscillations are observed at the beginning of the transient. To avoid these oscillations, it is advisable to use an appropriate filter.
3. Instead of acceleration feedback (or double differentiation of the displacement), feedback based on the difference in pressures in the chambers of the hydraulic cylinder can be used.

VI. SYMBOLS USED IN EQUATIONS

A_1, A_2	effective areas of the hydraulic cylinder	$[\text{m}^2]$
a	acceleration of the hydraulic cylinder rod	$[\text{m/s}^2]$
b	acceleration of the hydraulic cylinder rod	$[\text{m}]$
m	reduced mass of all moving parts to the axis of the hydraulic cylinder	$[\text{Kg}]$
C_S	elastic load constant	$[\text{N/m}]$

E	bulk modulus of elasticity of the fluid	[Pa]	V_{01}	Initial volumes in the left and right chambers of the hydraulic cylinder	[m ³]
f	coefficient of viscous friction	[Ns/m]	V_{02}		
F_{C0}	Coulomb dry friction force	[N]	x	Displacement of the servo distributor plunger	[m]
$F_C(v)$	Coulomb dry friction force	[N]	x_m	maximum plunger travel	[m]
$G(x)$	hydraulic conductivity	[m ³ /s.Pa ^{-1/2}]	y	moving the hydraulic cylinder rod	[m]
G_m	maximum hydraulic conductivity	[m ³ /s.Pa ^{-1/2}]	y_m	moving the hydraulic cylinder rod	[m]
i	current in the servo valve	[A]	φ	rotating the baffle in the servo valve	[rad]
K_a	amplifier gain	[-]	ρ	hydraulic oil density	[kg/m ³]
K_y	displacement feedback gain	[V/m/s]	μ	flow rate coefficient	[-]
$K_{\dot{y}}$	velocity feedback gain	[V/m/s ²]	ζ_f	damping coefficient of the barrier	[-]
$K_{\ddot{y}}$	acceleration feedback gain	[V/m]	ζ_0	damping coefficient of the drive system	
K_{vy}	speed gain coefficient of the electrohydraulic system	[s ⁻¹]	ω_0	natural frequency of the drive system	
k_M	scale factor	[-]			
$\bar{k}_{u\dot{x}}$	velocity feedback coefficient	[-]			
$\bar{k}_{u\ddot{x}}$	acceleration feedback coefficient	[-]			
K_o	conductivity coefficient	[-]			
k_i	current gain	[rad/A]			
k_U	gain of an electromechanical converter	[A/V]			
k_{rx}	mechanical feedback coefficient in the servo valve	[rad/m]			
$k_{\varphi x}$	rotation coefficient of the partition	[m/rad]			
p_1, p_2	pressures in the cylinder chambers	[Pa]			
p_S	supply pressure	[Pa]			
p_0	return pressure	[Pa]			
Q_1, Q_2	flow rates through the hydraulic cylinder	[m ³ /s]			
T_f	the barrier time constant	[s]			
T_i	time constant of electromechanical converter	[s]			
$T_{\varphi x}$	the time constant of the plunger	[s]			
U_y	displacement feedback voltage	[V]			
$U_{\dot{y}}$	speed feedback voltage	[V]			
$U_{\ddot{y}}$	acceleration feedback voltage	[V]			
U_P	voltage from the electronic amplifier	[V]			
U_v	input voltage	[V]			
ΔU	voltage from the electronic adder	[V]			
v_r	cylinder rod speed	[m/s]			
v_m	maximum rod speed	[m/s]			
$V_1(y)$	volumes in the left and right chambers of the hydraulic cylinder	[m ³]			
$V_2(y)$					

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