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Generalized Net Model of the E-Learning Process

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Abstract: **The paper presents a original model of the e-learning process, developed using the Generalized Net (GN) apparatus. The main stages in digital education are modeled - from access to learning resources in electronic platforms to ongoing control and final assessment. Through the GN-specific transitions, the logical connections and information flows between learners, teachers and learning management systems (LMS) are described. The model allows tracking individual progress and offers a theoretical basis for optimizing software architectures in the modern educational environment.**

Keywords: e-learning, learning process, generalized net, education.

I. Introduction

The rapid development of information technology in the 21st century has led to an increase in the application of e-learning in education. In modern traditional education, lessons are built containing one, two or several electronic applications.

Many digital learning opportunities are offered. There are platforms with open online courses - these are generally available online courses with open access MOOC (Massive Open Online Course), which are offered by various universities and organizations around the world. Video sharing websites have been built, which provide a service to users for uploading, sharing, watching videos and live streaming. Content management systems (LCMS - learning content management system) are applied - computer software that provides the ability to create, store, manage and distribute content. Artificial intelligence (AI) is also an assistant in learning, which assists learners with instructions for solving tasks, tests and analyses in all academic disciplines and personalizes the learning process, analyzing the data of learners from the performance of tasks. Gamification, virtual and augmented reality in education (VR, AR) are used, simulations are applied, and many other existing and developing digital tools are increasingly present and entering education in the modern world [7].

II. Materials and Methods

A model of the e-learning process was created, developed using the Generalized Networks (GN) apparatus and implemented using specialized software for creating generalized network models. The logical connections and information flows between students, teachers and learning management systems (LMS) were described using GN-specific transitions [1,2,4,6]

A flowchart of the e-learning process model was created using a graphical editor, logical connections and geometric figures, which implement the drawing of block structures [3].

The e-learning process model was tested using the GN IDE simulation environment to track the movement of the tokens and its operation. The simulation process was implemented using GN IDE (Generalized Nets Integrated Development Environment) and is an official software for simulating generalized network models [8, 9, 12, 13].

A model of teaching in the subject of informatics, including digital technologies, was applied.

The creation and testing of a new, more interesting and productive teaching method comes as a result of applied teaching in informatics with a variety of possible digital technologies with a group of students for a period of 4 months in a secondary school in Sofia in grade 12. A study was conducted between the results achieved in two groups of students after initial testing and proved almost the same result. The groups of 10 students were taught in different ways. One group used a programming environment and approved electronic textbooks for the subject. The other students used the same tools, as well as all possible digital technologies for the subject. As a result of the two different applied methods in teaching in informatics, the group using various digital technologies showed greater interest in learning, mastered the material faster and showed higher results. In the last testing in the study, the two groups showed a different result and a difference in achievements of about one unit.

A generalized network model of the e-learning process was created and proposed after research and review of

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various materials in the field of generalized networks and learning [10, 11, 14, 15].

The e-learning process model structures the relationship between all participants through electronic platforms and contains:

1. Participants in the system:

- Teachers (mentors and designees) who set a lesson by creating learning content, setting tests and assignments and moderating the system;
- Students, learning part of the time independently, go through the learning content and provide feedback;
- Groups of students who work on projects, solve tasks and tests and teach each other.

2. Digital tools:

- Electronic platforms, such as LMS (Learning Management Systems), which store resources, conduct lectures, discussions, tests, and track student progress;
- Collaboration tools - Google Docs, Google Slides and tools for checking knowledge through gamification.

3. Model process. It takes place in the following stages:

Stage 1: Preparation of the platform and initial work of students

- The teacher prepares training materials and takes care of the design and moderates the system;
- Students work with the materials, including part of the time independently.

Stage 2: Collaborative work and analysis of student knowledge by the teacher

- Active learning through discussions of the material and tasks in Teams and Zoom in real time.
- Checking students' knowledge for an initial level of content by the teacher through surveys and tests prepared by him.

Stage 3**:** Student work in groups

- Groups of students work in a shared product and are present in a virtual classroom;
- The teacher monitors the work of the students and provides consultations.

Stage 4**:** Assessment and feedback

Groups share the results of the tasks through the screen of the digital device

- Through a test test in the electronic platform, the students' knowledge is assessed;
- The teacher monitors and analyzes the data from the group training to help with the students' difficulties
- A final assessment of the students' knowledge is made and a decision is made to move to the next module or study the material again.

Before creating a general network model of the e-learning process, we show the logic and connections between objects through a flowchart based on the stages through which learning occurs according to the presented model (Fig. 1) [3,5]

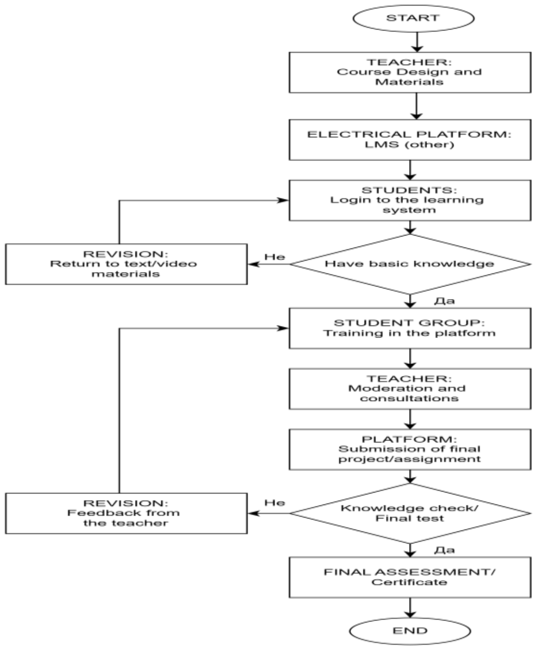

Fig. 1. Flowchart of an e-learning process model

Based on the description of the e-learning process model and as a continuation of the flowchart, its generalized network model was created [1,2,4,6] (Fig. 2).

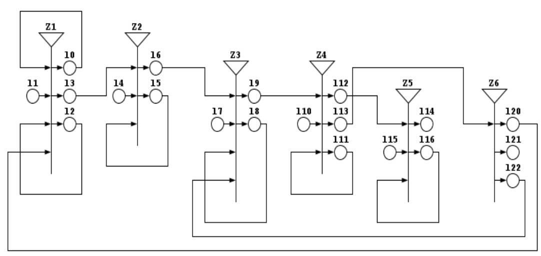

Fig. 2. Generalized net model of the e-learning process

The generalized network E contains the set of transitions $E = \{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6\}$, where the transitions describe the following processes

The following tokens enter the OM network:

α – basic tokens and represent the students (learners)

- The tokens enter the model through position l_1 at the beginning of the scheme.

- The tokens accumulate in the review and feedback positions in the positions: l_2 – design; l_5 – materials; l_8 – test; l_{11} – task; l_{16} – certificate.

The tokens are split at position l_{13} of the transition Z_6. The token is divided into logical copies at positions l_{21} – ends (discards), l_{22} – restart of the test, l_{20} – restart of the design.

- **The tokens merge** in the transition Z_1 – Here the tokens from the starting position l1 and those returning for correction from l_{20} are merged in l_3 to start a new learning cycle (again).

- **The tokens merge** in the transition Z_3 – Here the tokens of the new students in position l_6 and those returning for correction of the test from l_{22} are collected in a common group for evaluation.

- **Token movement** - Tokens move in positions l_1-> l_3 ->l_5-> l_9 ->l_{12} >l_{14}. Positions l_{20} and l_{22} are critical because they return the tokens to an earlier stage in the OM model to guarantee the quality of learning.

β – tokens – Tokens that enter positions l_2 l_4 l_7 l_{10} l_{15} carry resources. These are **information tokens** that carry materials and tests and merge with the main token of the student in the corresponding transitions.

γ – tokens – The tokens of the teachers – They create materials and play the role of controllers and evaluators in the OM model. They move in the feedback and control positions – l_0, l_{11}, l_{15} and l_{16}. The γ token enters through position l_0 and initiates the learning process. In l_{11} the teacher enters to help the students solve the task of learning the material and correcting errors. In positions l_{15} the teachers check and review the results of the training. In l_{16} administrative control is carried out when issuing documents.

In the model we have a conditional transition. The tokens receive the characteristics "test result", "certificate". If the result of any logical condition is false the token remains in position l_2,l_5,l_8, or returns to the initial position l_1 through Z_6, until the requirements for true are met.

Transition Z1: Preparation and design
Start of training

$$Z_1 = <\{l_1, l_2, l_{20}\}, \{l_3, l_2\}, r_1>$$

The tokens α enter through position l_1 or return from l_{20} for design correction.

The token γ enters through position l_0 and initializes the training process and remains in the system for creating materials, control and monitoring.

$$r_1 = \begin{array}{c|ccc} & l_0 & l_3 & l_2 \\ \hline l_0 & W_{0,0} & \text{False} & \text{False} \\ l_1 & \text{False} & W_{1,3} & W_{1,2} \\ l_2 & \text{False} & W_{1,3} & \text{true} \\ l_{20} & \text{False} & W_{1,3} & W_{1,3} \end{array}$$

$W_{1,3}$ = “There is a ready-made course design.”
$W_{1,2}$ = “Additional training available”

Transition Z2: Training and materials

$$Z_2 = <\{l_3, l_4, l_5\}, \{l_6, l_5\}, r_2>$$

$$r_2 = \begin{array}{c|cc} & l_6 & l_5 \\ \hline l_3 & W_{3,6} & W_{3,6} \\ l_4 & \text{false} & \text{true} \\ l_5 & W_{3,6} & \text{true} \end{array}$$

$W_{3,6}$ = “Have the learning materials been mastered”

Transition Z3: Testing and Evaluation

$$Z_3 = <\{l_6, l_7, l_8, l_{22}\}, \{l_9, l_8\}, r_3>$$

$$r_3 = \begin{array}{c|cc} & l_9 & l_8 \\ \hline l_6 & W_{6,9} & W_{6,9} \\ l_7 & \text{false} & \text{true} \\ l_8 & W_{6,9} & \text{true} \\ l_{22} & W_{6,9} & W_{6,9} \end{array}$$

$W_{6,9}$ = “Entrance test passed”

Transition Z4: Solving a final assignment and feedback

$$Z_4 = <\{l_9, l_{10}, l_{11}\}, \{l_{12}, l_{13}, l_{11}\}, r_4>$$

$$r_4 = \begin{array}{c|ccc} & l_{12} & l_{13} & l_{11} \\ \hline l_9 & W_{9,12} & W_{9,13} & (W_{9,12}\ W_{9,13}) \\ l_{10} & \text{false} & \text{false} & \text{true} \\ l_{11} & W_{9,12} & W_{9,13} & \text{true} \end{array}$$

$W_{9,12}$ = "The task has been successfully solved"
$W_{9,13}$ = “Teacher assistance/correction needed”

Transition Z5: Certification

$Z_5 = <\{l_{12}, l_{15}, l_{16}\}, \{l_{14}, l_{16}\}, r_5>$

$$r_5 = \begin{array}{c|cc} & l_{14} & l_{16} \\ \hline l_{12} & W_{12,14} & W_{12,14} \\ l_{15} & \text{false} & \text{true} \\ l_{16} & W_{12,14} & \text{true} \end{array}()$$

$W_{12,14}$ ="Certificate issued successfully"

Transition Z_6: Feedback and restart

$Z_6 = <\{l_{13}\}, \{l_{20}, l_{21}, l_{22}\}, r_6>$

$$r_6 = \begin{array}{c|ccc} & l_{20} & l_{21} & l_{22} \\ \hline l_{13} & W_{13,20} & W_{13,21} & W_{13,22} \end{array}()$$

$W_{13,20}$ = “Return to design”

$W_{13,21}$ = “The student has exhausted the number of attempts or is giving up - the end”

$W_{13,22}$ = “Return to test”

Testing the model in the GN IDE simulation environment A simulation was performed on the general network model of the e-learning process to track the movement of the tokens and its operation. The simulation process was carried out using GN IDE (Generalized Nets Integrated Development Environment) and is an official software for simulation of generalized network models [8, 9]. Figures 3. and 4. show the movements of the α–tokens in the first transition of the e-learning process model, and Fig. 5. shows screens from the last stage of the movement of the tokens in the network. (Figure 3, Figure 4, Figure 5)

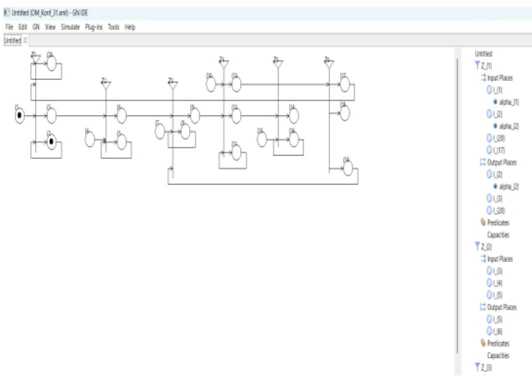

Fig. 3. Entry and movement of the α-token in the initial transition of the OM model.

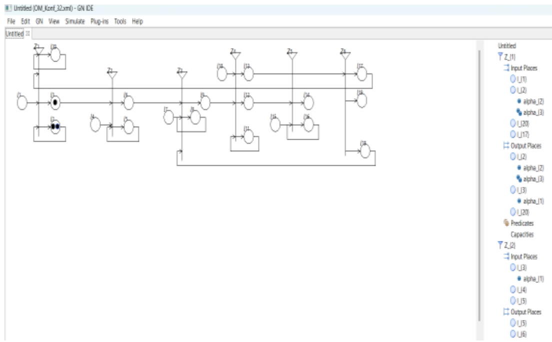

Fig. 4. Movement of the α-token in transition Z_1.

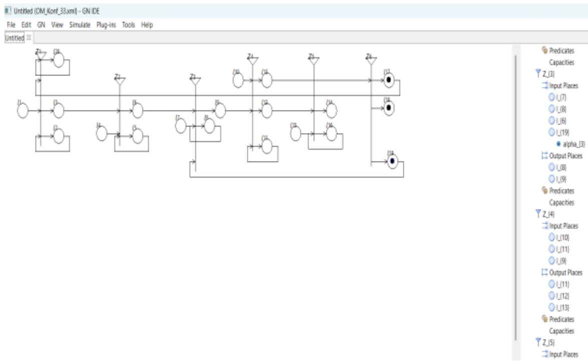

Fig. 5. Movement and realization of the α-token (learner) when using the e-learning process model

III. Results and Discussion

The generalized net model of the e-learning process is up-to-date and effective. The training system monitors and automatically returns the student to a certain previous position without the intervention of the teacher (from positions l_{20}, l_{22}) for knowledge correction. The model offers personalization and tracking, by monitoring and deciding whether the student has the necessary knowledge for the next level through the predicates $W_{i,j}$. The OM model effectively manages resources through individual student and material tokens and allows updating of the learning content without interrupting the current learning of the students. The model tracks and analyzes the learning, progress and gaps (accumulation in l_8 and l_{11}) of the students thanks to the fact that each passage through a position leaves a characteristic.

IV. Conclusions

The OM model of the e-learning process is valuable with its tracking and personalization in learning, effective management of learning resources and feedback automation.

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