Secure Systems Design and Technology Development. Veliko Tarnovo, Bulgaria
Proceedings of the International Scientific Conference, 2026. Volume II, 155-158

Optimization of Statistical Analysis for Gear Inspection

Kiril Lengerov
Faculty of Mechanical Engineering
Department: Mechanical and Instrument Engineering
Technical University of Sofia,
Branch Plovdiv
Plovdiv, Bulgaria
k.lengerov@tu-plovdiv.bg

Ivan Tashev
Faculty of Mechanical Engineering
Department: Mechanical and Instrument Engineering
Technical University of Sofia,
Branch Plovdiv
Plovdiv, Bulgaria
ivan.tashev@tu-plovdiv.bg

***Abstract*— This paper presents a method for evaluating the machining accuracy and process stability of the gear cutting and gear hobbing of cylindrical gears. Through statistical analysis and the use of mathematical evaluation criteria, the stability of the machining process has been established. This makes it possible to determine the optimal setup parameters and predict the maximum number of parts that can be produced with the initial setup of the machining system.**

***Keywords*— manufacturing accuracy, process stability, statistical analysis.**

I. Introduction

The manufacture of gears by gear cutting and gear hobbing is widespread in mechanical engineering. To assess the process accuracy, it is advisable to monitor the length of the common normal W_k and the tooth thickness along the constant chord S_c. The preferred parameter is the length of the common normal, which reflects both the kinematic accuracy and the lateral clearance in the gear meshing.

To ensure the reliability of the inspection results, the accuracy of the machine, equipment, and workpieces must be within the specified limits for the corresponding accuracy class of the gear wheels being manufactured.

Through statistical analysis, the type of manufacturing process is identified, the target operating value is determined, and the control limits within which the mean value of the control parameter may vary are established; the progress of the technological process is monitored, and the moment is determined at which the technological system should be readjusted or the tool should be changed [1], [2] [3].

II. Materials and Methods

The input data for the analysis is obtained by inspecting all parts produced during a single setup of the machining system. This is necessary because the tool life, expressed as the number of parts produced during a single setup, is short. The inspection of the parts is performed in a sequence corresponding to the order in which they were manufactured.

To obtain a visual representation of the course of the manufacturing process, a scatter plot is constructed based on the inspection data. In cases where it has the form shown in Fig. 1, the hypothesis can be made that the process is statistically stable, and when it has the form shown in Fig. 2, that the process is technologically stable.

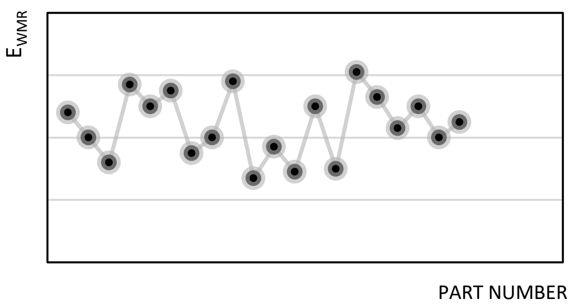

Fig. 1. A scatter plot illustrating a statistically stable process.

Online ISSN 3044-7771
https://doi.org/10.68302/std2026.vol2.129

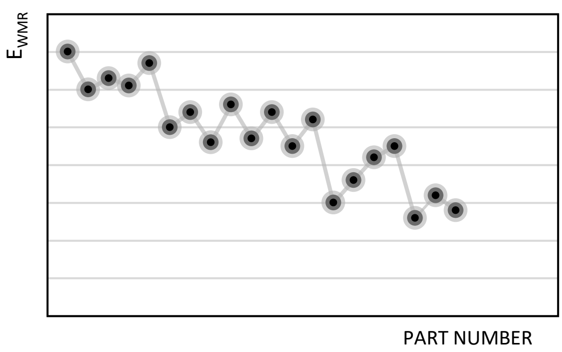

Fig. 2. A scatter plot illustrating a technologically unstable process.

In both cases, it is advisable to demonstrate the stability of the process using a uniform methodology. To simplify the analysis, the details are grouped into small samples consisting of 3 to 7 gear wheels, which are processed sequentially. The sample size is determined by the following condition:

$$3 \leq n \leq N/4 \quad (1)$$

where n is the number of gears in the sample;

N – the total number of gears produced in a single setup.

When $N<12$, the specified method cannot be applied.

For each sample, the following statistical estimates are calculated:

$E_{W_{ms}}$ – mean value of the deviation from the nominal length of the general normal.

$$E_{Wms} = \frac{\sum_{i=1}^{n} E_{Wmr}}{n} \quad (2)$$

$$S^2 = \frac{\sum_{i=1}^{n}(E_{Wmri} - E_{Wms})^2}{n-1} \quad (3)$$

If the hypothesis of statistical stability of a technological process is being tested, the homogeneity of variances and the stability of the mean values for the conditional samples are verified. For this purpose, the Cochran criterion $G_{max} < G_\alpha, K_G$ and the Student's criterion $t < t_\alpha, K_t$. The tabulated values of the criteria $G\alpha$, KG, and $t\alpha$, Kt are determined based on the accepted significance level and the degrees of freedom: $K_G = n - 1$ and $K_t = 2\ (n - 1)$. The values of the criteria for the manufactured parts are determined by the following expressions [4]:

$$G_{max} = \frac{S_{max}^2}{\sum_{j=1}^{m} S_j^2} \quad (4)$$

$$t = \sqrt{n}\frac{|E_{Wmr1} - E_{Wmr2}|}{\sqrt{S_1^2 + S_2^2}} \quad (5)$$

where m is the total number of conditional samples;

s_{max}^2 is the maximum variance across the set of samples under consideration.

The G_{max} criterion is determined for the entire set of samples, while the t criterion is applied sequentially to pairs of samples.

If the criteria of William Cochran and Student are satisfied for more than *90%* of the samples, the process is considered statistically stable. In this case, the characteristics of the theoretical accuracy diagram are determined:

$\bar{\bar{E}}_{Wmr}$ - Average value

$$\bar{\bar{E}}_{Wmr} = \frac{\sum_{j=1}^{m-p} \bar{E}_{Wmri}}{m-p} \quad (6)$$

$\bar{s}^2$ - average variance

$$\bar{s}^2 = \frac{\sum_{j=1}^{n(m-p)}\left(\bar{E}_{Wmri} - \bar{\bar{E}}_{Wmr}\right)^2}{n(m-p)-1} \quad (7)$$

ω - instantaneous scattering field

$$\omega = 2.K_w.\bar{s} \quad (8)$$

Where p is the number of samples excluded from the analysis because they did not meet the Cochran or Student criteria

K_w - a coefficient that determines the practical limits of scattering.

If the hypothesis of technological stability of the process is accepted, the homogeneity of the variances of the conditional samples is tested using Cochran's criterion. Next, the linear regression equation for the mean value of the deviation from the length of the grand normal is determined, and its objectivity is tested.

The regression equation is of the form:

$$\bar{E}_{OWmrj} = a + b.\tau_j \quad (9)$$

Where $\bar{E}_{OW_{mrj}}$ is the mean value of the *jth* sample, determined by the regression equation;

τ_j – the sample number;

a and b – the statistical coefficients

$$a = \frac{\sum_{j=1}^{m} \bar{E}_{Wmrj}}{m} - b\tau \quad (10)$$

$$b = \frac{\sum_{j=1}^{m} \bar{E}_{Wmrj}\left(\tau_j - \bar{\tau}\right)}{\sum_{j=1}^{m} \tau_j} \quad (11)$$

$$\bar{\tau} = \frac{\sum_{j=1}^{m} \tau_j}{m} \quad (12)$$

The validity of the regression equation is tested using Fisher's criterion:

$$F = \frac{s_2^2}{s_1^2} \leq F\alpha_{0,K_1,K_2,} \quad (13)$$

Where: $F\alpha_0$, K_1, K_2 is the table value of the test statistic at a significance level of α and degrees of freedom $K_1 = m - 2$ and $K_2 = m\ (n - 1)$

$$s_2^2 = \frac{n.\sum_{j=1}^{m}\left(\bar{E}_{Wmrj} - \bar{E}_{OWmrj}\right)^2}{m-2} \tag{14}$$

$$s_1^2 = \frac{\sum_{j=1}^{m} s_j^2}{m} \tag{15}$$

If the homogeneity of the variances is proven and the regression equation is adequate, the process is considered technologically stable. Its parameters are:

- The regression equation for $\bar{E}_{OW_{mrj}}$
- The mean variance

$$\bar{s}^2 = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(E_{Wmrij} - \bar{\Sigma}_{owmrj}\right)^2}{n(m-p)-1} \tag{16}$$

- The instantaneous scattering field

$$\omega = 2.K_w.\bar{s} \tag{17}$$

Once the stability of the process has been demonstrated, its accuracy and calibration can be analysed.

The accuracy coefficient K_T is used to evaluate accuracy.

$$K_T = \frac{W_\Sigma}{T_w} \tag{18}$$

where ω_Σ is the total scattering field resulting from the combined effect of systematic and random factors;

T_w – the tolerance for the length of the common normal.

For a statistically stable process:

$$\omega_\Sigma = \omega \tag{19}$$

For technologically stable processes:

$$\omega_\Sigma = \sqrt{\omega^2 + 3b^2\left(\frac{N}{n} - 1\right)^2} \tag{20}$$

The accuracy coefficient must be within the following range:

$$0{,}75 \le K_T \le 0{,}85 \tag{21}$$

When $K_T > 0{,}85$, the process does not meet the accuracy requirements.

When $K_T < 0{,}75$, there is an excess margin of accuracy. In this case, cutting conditions can be intensified, or, for technologically stable processes, more parts can be machined with a single dimensional setting. Their number N_{max} will be:

$$N_{max} = 1 + n\sqrt{\frac{(0{,}85.T_{wm})^2 - \omega^2}{3b^2}} \tag{22}$$

The tuning of the technological process is evaluated using the tuning coefficient K_H:

$$K_H = \frac{\varepsilon}{T_{Wm}} \tag{23}$$

Where: ε is the tuning deviation.

For statistically stable processes:

$$\varepsilon = \left|\bar{\bar{E}}_{Wmc} - \frac{E_{Wms} + E_{Wmi}}{2}\right| \tag{24}$$

And in the case of technologically stable processes:

$$\varepsilon = \left|\bar{E}_{OWmr1} - E_{Wadj.}\right| \tag{25}$$

Here, $E_{W_{ms}}$ and $E_{W_{mi}}$ are the tolerance limits for the length of the common normal.

$E_{W_{adj.}}$ – the set value for the deviation from the length of the common normal.

When adjusting the length of the common normal during the manufacturing process:

$$E_{Wadj.} = E_{wmi} + \omega_{mes} + K_W.\bar{s} \tag{26}$$

and with a downward trend

$$E_{Wadj.} = E_{wms} - \omega_{mes.} - K_W.\bar{s} \tag{27}$$

Where: $\omega_{mes.}$ is the measurement error in gear inspection.

The manufacturing system is well-adjusted if $K_H \le 0.1$

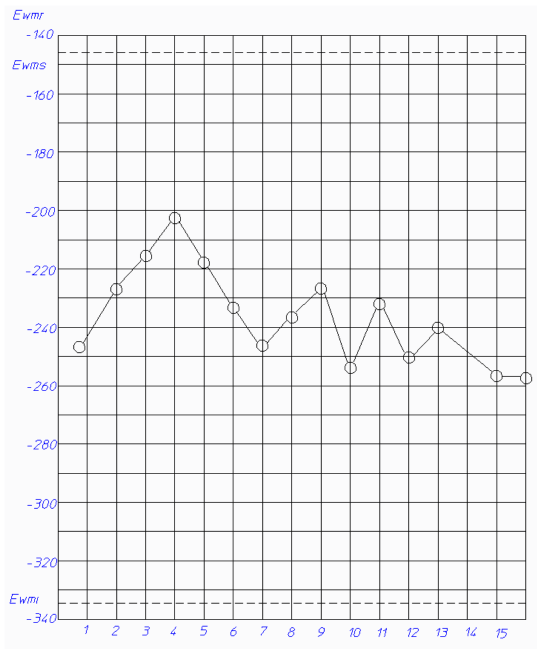

Fig. 3. A scatter plot based on an analysis of the gear-cutting process for a cylindrical gear with straight teeth

The applicability of the described methodology has been verified under production conditions. As an example, one can cite the results of an analysis of the manufacturing process for gear cutting of a cylindrical gear with straight teeth. A scatter plot was constructed based on the inspection results, shown in Fig. 3. The nature of the scatter plot provides grounds for hypothesizing the technological stability of the process. The machined parts were divided into *5* conditional samples with a size of $n = 3$ parts. The mean values $\bar{E}_{W_{mrij}}$ and variances for all samples were calculated. The Cochran criterion has a value of $G_{max} = 0.2935$. The tabulated value is $G_{0.05};\ z = 0.6838$. Since $G_{max} < G_{\alpha},\ K_G$, the variances are homogeneous [5]. The regression equation is of the form:

$$\bar{E}_{OWmrj} = 214{,}5 - 7\tau_j \tag{28}$$

In this case, $s_1^2 = 234{,}07$ and $s_2^2 = 191{,}32$. The Fisher's F-statistic is $F = 1.2234$. Its critical value is $F_{0.05;\ 3;\ 10} = 3.41$. Since $F < F_{\alpha,\ K1,\ K2}$, the regression equation is adequate.

The checks performed show that the process is technologically stable. Its mean variance is $S^2 = 159.87$. The dispersion range, with a *99.73%* probability that the part dimensions will fall within the specified limits and with $N = 15$ measured parts, will be: $\omega = 2.4{,}747.\sqrt{159{,}87} = 120\mu m.$

The total scattering cross section for the process with $N = 15$ particles, $\omega = 120\ \mu m$, $b = -7$, and $n = 3$ is:

$$\omega_\Sigma = 129{,}43\ \mu\text{m} \tag{29}$$

The adjustment coefficient will be:

$$K_H = \frac{129{,}43}{190} = 0{,}68 \tag{30}$$

Since the process has a margin of accuracy, it is possible to fine-tune or replace the tool after machining a larger number of parts. This number is $N_{max} = 27$ parts.

The adjustment value of the deviation from the length of the common normal at $\omega_{mes.} = 19$ µm is $E_{adj.} = -224$ µm. The theoretical value of the deviation for the first sample is obtained from the regression equation $E_{OW_{mri}} = -221{,}5\mu m$, and the adjustment offset is $\varepsilon = 2.5\ \mu m$.

The adjustment coefficient is $K_H = 2.5/190 = 0.013$, which indicates very good process adjustment.

III. Conclusion

A methodology for standardized statistical analysis that enables precise control over manufacturing accuracy in the production of gear wheels. By applying mathematical stability criteria, the optimal time for machine readjustment or tool replacement is determined, thereby preventing the occurrence of defects. The main benefit of the analysis is the ability to objectively and thoroughly assess precision, which allows for an increase in the number of parts processed per setup. This leads to more efficient use of resources and optimization of production modes without compromising the quality of the gear meshing. The implementation of this approach ensures high economic efficiency and stability of technological processes in mechanical engineering.

Acknowledgments

The author/s would like to thank the Research and Development Sector at the Technical University of Sofia for the financial support.

REFERENCES

[1] V.Georgieev, Pashov St., Mechanical engineering technology, fundamentals of mechanical engineering technology, 2003.

[2] V.Georiev, A. Lengerov, Quality Control and Management, First Edition, ISBN-978-619-7010-23-7, 2014.

[3] Youssef H., H. El-Hofy, M. Ahmed, Manufacturing Technology Materials, Processes, and Equipment, ISBN 9781032432489, Boca Raton, Florida, CRC Press, 2023.

[4] Kyratsis P., J. P. Davim, Experiments and Simulations in Advanced Manufacturing, ISBN 978-3-030-69471-5, eBook ISBN 978-3-030-69472-2, Springer Cham, 2021, DOI https://doi.org/10.1007/978-3-030-69472-2

[5] Gillespie LaRoux, Design for Advanced Manufacturing: Technologies and Processes, ISBN 9781259587450, New York City, McGraw Hill, 2017.