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Improving the Accuracy of Gear Transmission by Dividing the Kinematic Error

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***Abstract*—This paper discusses a method for improving the kinematic accuracy of gear trains through precise analysis and decomposition of the errors of individual gears during assembly. Through mathematical modeling and analysis of collected data, the optimal meshing of the gear wheels is determined, allowing for the minimization of the total error.**

Keywords— angle of rotation, gear transmission, kinematic accuracy, kinematic error.

I. Introduction

The kinematic accuracy of gear trains determines the ability of each gear to transmit rotational motion with the closest possible match between the theoretical and actual angles of rotation of the driven gear.

In a kinematic pair consisting of two gears with satisfactory accuracy, the transmission in which these gears operate may exhibit unsatisfactory accuracy. In many cases, a transmission containing two gears with satisfactory accuracy may exhibit satisfactory kinematic accuracy.

Such a comparison shows us that to ensure high kinematic accuracy of mechanisms, it is necessary to solve the complex problem of precise control both during the manufacturing stage of their components and during the assembly stage [1]. If, during the manufacturing stage, the accuracy of the gears can be guaranteed by adhering to the requirements of the manufacturing process, then the assembly stage may involve the procedure for selecting a suitable kinematic pair [2][3].

II. Materials and Methods

When designing gear transmissions, the assembly stage can be described and summarized in two procedures:

- Analysis of the existing pair to determine its compliance with accuracy requirements;
- Synthesis of a satisfactory pair based on the results of the analysis.

The analysis procedure involves decomposing the contributions to the total kinematic error of the driving and driven links. Using this method, the decomposition can be as follows.

It is known that the kinematic error of the pair is the difference between the actual angle of rotation of the driven link θ_{act} and the nominal angle θ_{nom}.

$$\Delta\theta = \theta_{act} - \theta_{nom} \tag{1}$$

If we record the kinematic error of the pair over several revolutions of the driven link, defining the complete cycle of tooth engagement between the driving and driven links, then the number of revolutions of these links will be equal to n_1 and n_2, respectively to (2) (3).

$$n_1 . z_1 = n_2 . z_2 \tag{2}$$

$$n_1 = \frac{z_2}{z}, n_2 = \frac{z_1}{z} \tag{3}$$

where z_1 and z_2 are the number of teeth on the driving and driven gears, respectively, and z is the greatest common divisor of z_1 and z_2.

Fig. 1 shows a case where $z_1 = 12$ and $z_2 = 16$: $z = 4$, $n_1 = 4$, $n_2 = 3$, and the periods T_1 and T_2 correspond to the rotational speeds of the driving and driven links, respectively. The total number of measurements in this case will be $n_2(P/R)$, where *P/R* is the number of readings from the output shaft rotation transducer (Pulses per Revolution).

Further considerations are based on the obvious property of kinematic error, namely that for a centered

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kinematic error, the mathematical expectation is equal to zero (4).

$$m(\Delta\theta) = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}(\Delta\theta_i - \Delta\theta^0) = 0 \qquad (4)$$

where $\Delta\theta^o$ is the initial (central) value of $\Delta\theta$.

Let us consider a specific point A on the active side surface of the drive wheel (Fig. 1). The moment of its contact with the teeth of the driving gear during the first revolution of the driven gear is characterized by point 1 and the corresponding kinematic error caused by the combined influence of errors in the driving and driven members. The moment of contact for the second rotation corresponds to point 2, and for the third rotation — to point 3, and so on. Let us consider the average value of the kinematic error $\Delta\theta$ for all contact points separated from one another by a distance T_2 (or by a number M) [4][5].

$$\frac{1}{\mathrm{n}_2}\sum_{\mathrm{i}=1}^{\mathrm{n}_2}\Delta\theta.(k+(\mathrm{i}-1).\mathrm{M}) \qquad (5)$$

where k is the number of the measured kinematic error corresponding to the position of point A within the first period T_2 ($k = 1 \ldots M$), where M is the number of revolutions measured for the output shaft (corresponding to the period T_2).

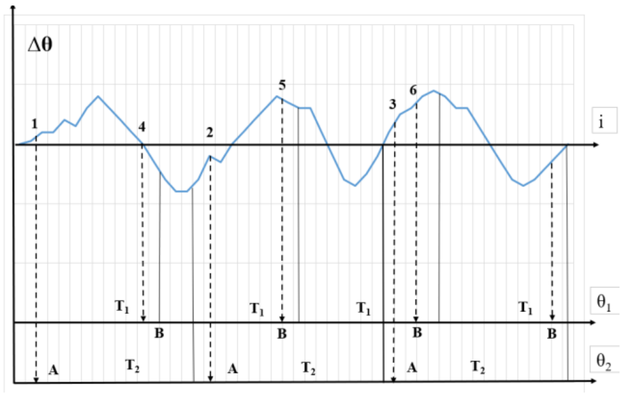

Fig. 1. Diagram illustrating the separation method

Equation (6), taking into account the contributions of the errors of the driving wheel $\Delta\theta_1$ and the driven wheel $\Delta\theta_2$, will take the form.

$$\frac{1}{\mathrm{n}_2}\sum_{\mathrm{i}=1}^{\mathrm{n}_2}\Delta\theta.(k+(\mathrm{i}-1).\mathrm{M}) \qquad (6)$$

Since the contribution of the driven wheel $\Delta\theta_2$ at point A is a constant value, then

$$\frac{1}{n_2}\sum \Delta\theta_2\,(r) = \Delta\theta(A) \qquad (7)$$

The contribution of the drive wheel error is a random variable (since point A touches different points on the drive wheel). Therefore, taking (5) into account

$$\frac{1}{n_2}\sum \Delta\theta_1(r) \approx 0 \qquad (8)$$

Taking expressions (6) and (8) into account, relation (5):

$$\Delta\theta_2(A) = \Delta\theta(k_A) \approx \frac{1}{n_2}\sum_{i=1}^{n_2}\Delta\theta(k_A + (i-1)M) \qquad (9)$$

In this way, the described procedure segments the initial data and calculates the kinematic error as the average value across the segments, applying the synchronous accumulation procedure.

If we repeat the calculations for segmenting the initial data by period T_1 (i.e., by the rotation period of the drive shaft (z_1/z_2M) for a specific point B on the static surface of the drive wheel (Fig. 1)), we obtain the following averaging formula:

$$\frac{1}{n_1}\sum_{j=1}^{n_1}\Delta\theta\left(k_B + (j-1)\frac{z_1}{z_2}M\right) \qquad (10)$$

where k_B is the kinematic error count corresponding to the position of point B within the first period T_1 ($k_B = 1 \ldots (z_1/z_2.M)$). An expression similar to (6) is also valid for this case, but in this form

$$\frac{1}{n_1}\sum \theta_1(r_1) = \Delta\theta(B) \qquad (11)$$

$$\frac{1}{n_1}\sum \theta_2(r_1) \approx 0 \qquad (12)$$

$$\Delta\theta_1(B) = \Delta\theta(k_B) \approx \frac{1}{n_1}\sum_{i=1}^{n_1}\left(k_B + (j-1)\frac{z_1}{z_2}M\right) \qquad (13)$$

Equations (9) and (13) allow us to decompose the errors of the driving and driven wheels at each kinematic error recording point ($1 \ldots M$) [6][7]:

$$\Delta\theta(\mathrm{k}) \approx \Delta\theta(\mathrm{k_A} = \mathrm{k}) + \Delta\theta(\mathrm{k_B} = \mathrm{k}) \qquad (14)$$

Using (14), it is possible to convert from angular to linear units and thus model the influence of individual gear parameters on its kinematic error. Equation (14) allows for the analysis of the influence of gear mounting on their kinematic error. From the diagram in Fig. 1, it is evident that if, for two gears with $z_1 \neq z_2$, the greatest common divisor is $z > 1$ (i.e., z_1 and z_2 are not prime numbers), then contact at point A occurs only with the fixed points of the driving gear. Therefore, by "rearranging" the driving gear z_2 times, each time shifting it by 1 gear tooth — which corresponds to a phase shift of the kinematic error by a factor of M/z_2 — it is possible to find a position where the kinematic error will have a nominal value, i.e., to minimize the function

$$\sum_i \sum_j |\Delta\theta_2(i,j)| \rightarrow min \tag{15}$$

The same objective can be achieved by changing the distances between the wheel centres. The use of (14) eliminates the need for physical experiments to determine the optimal wheel mounting and, based on the synthesis of the kinematic error, immediately determines the optimal contact teeth of the pair and/or the distance between the centres.

The above considerations can be extended to the case of a multi-stage transmission. An example is a two-stage transmission (Fig. 2), for which the following relationships hold:

$$\Delta\theta_{22}^{\Sigma}(i) = \Delta\theta'_{22}(i) + \Delta\theta'_{21}(i) + \Delta\theta'_{12}(i) + \Delta\theta'_{11}(i) \tag{16}$$

where $\Delta\theta_{22}^{\Sigma}(i)$ is the kinematic error of the driven shaft; $\Delta\theta'_{ij}(i)$ are the components of the kinematic error of the driven shaft corresponding to the influence of the intermediate gear *ij*.

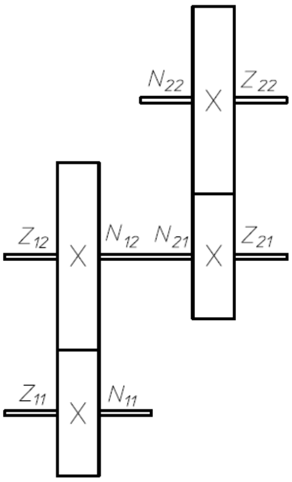

Fig. 2. Diagram of a two-stage transmission

By analogy with (9) and (13), it can be shown that

$$\Delta\theta'_{22}(i) = \frac{1}{n_{22}} \sum_{k=0}^{n_{22}-1} \Delta\theta_{22}^{\Sigma}\,(i + kM) \tag{17}$$

$$\Delta\theta_{21}^{\Sigma}(i) = \Delta\theta'_{21}(i) + \Delta\theta'_{12}(i) + \Delta\theta'_{11}(i) = \Delta\theta_{22}^{\Sigma}(i) - \Delta\theta'_{22} \tag{18}$$

$$\Delta\theta'_{21}(i) = \frac{1}{n_{21}} \sum_{k=0}^{n_{21}-1} \Delta\theta_{21}^{\Sigma}\left(i + k\frac{n_{22}}{n_{21}}M\right) \tag{19}$$

$$\Delta\theta_{12}^{\Sigma}(i) = \Delta\theta'_{12}(i) + \Delta\theta'_{11}(i) = \Delta\theta_{21}^{\Sigma}(i) - \Delta\theta'_{21}(i) \tag{20}$$

$$\Delta\theta'_{12}(i) = \frac{1}{n_{12}} \sum_{k=0}^{n_{12}-1} \Delta\theta_{12}^{\Sigma}\left(i + k\frac{n_{22}}{n_{12}}M\right) \tag{21}$$

$$\theta_{11}^{\Sigma}(i) = \Delta\theta_{12}^{\Sigma}(i) - \Delta\theta'_{12}(i) = \Delta\theta'_{11}(i) \tag{22}$$

If it is necessary to reduce the kinematic error of the *ij* element to its pitch diameter, i.e., to $\Delta\theta ij$, the transmission ratio between the *ij* component and the end link must be taken into account according to the formula:

$$\Delta\theta_{ij}^{\Sigma}(k) = \Delta\theta_{ij}(k)U_i \tag{23}$$

from which it follows that

$$\Delta\theta_{22}(k) = \Delta\theta'_{22}(k) \tag{24}$$

$$\Delta\theta_{21}(k) = \Delta\theta'_{21}(k).U_2 \tag{25}$$

$$\Delta\theta_{12}(k) = \Delta\theta'_{12}(k).U_2 \tag{26}$$

$$\Delta\theta_{11}(k) = \Delta\theta'_{11}(k).U_1U_2 \tag{27}$$

Unfortunately, these formulas can be used in practice only if the $\Delta\theta_{22}^{\Sigma}$ measurements are performed using a sufficiently accurate angular displacement transducer and the following condition is met:

$$\Delta\theta_{ij}^{\Sigma}(k) > \Delta\omega \tag{28}$$

where Δω is the absolute error of the transducer [8].

III. Conclusion

This article presents a theoretically grounded methodology for analyzing and decomposing the components of kinematic error in gear trains by applying the synchronous accumulation procedure. Based on the conducted research and the derived mathematical relationships, the following conclusions can be drawn:

- The proposed method allows for precise segmentation of the initial data and identification of the individual contribution of each separate component (driving and driven) to the total kinematic error. This eliminates the need for complex physical experiments to optimize the assembly.
- By mathematically modeling the phase shift of the error (through "rearrangement" of the teeth or changing the center distance), conditions are created to achieve minimum values of the kinematic error as early as the design and assembly stages.
- It has been established that the method is applicable with high accuracy for single-stage and two-stage mechanisms. For multi-stage transmissions (three or more stages), the accuracy of the calculations progressively decreases, which limits the practical application of the method to only the last one or two stages of the transmission.
- To ensure reliable results, it is critical to use high-precision angular displacement transducers (with a minimum of 2,500 counts per revolution),

whose inherent absolute error is smaller than the kinematic error of the components under investigation.

In conclusion, the described approach represents a reliable tool for the precise control and synthesis of kinematic pairs, ensuring high transmission accuracy while adhering to the defined design constraints.

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